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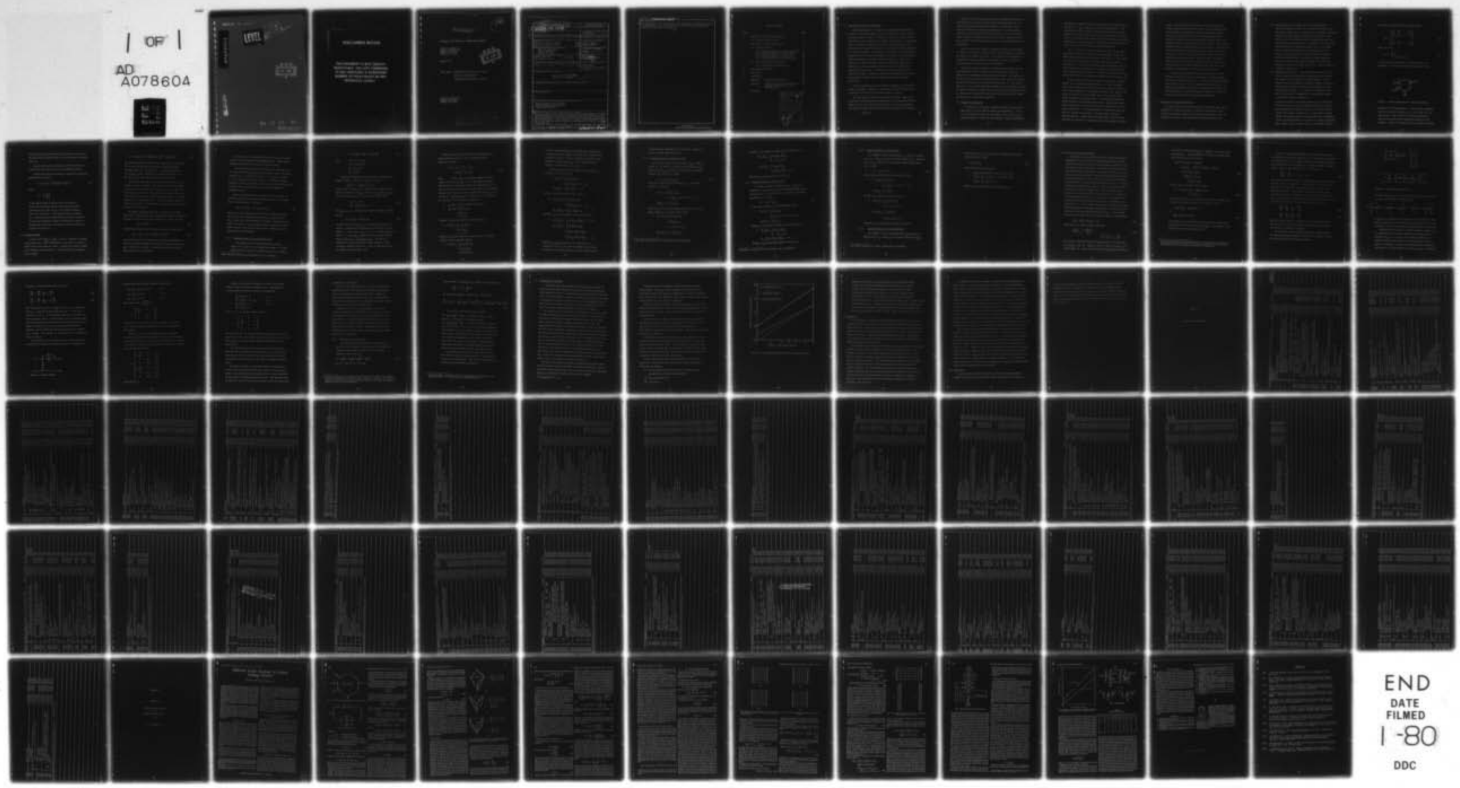

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EFFICIENT FAULT ANALYSIS IN LINEAR ANALOG CIRCUITS

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This report addresses the problem of high-resolution fault diagnosis in linear, analog electrical circuits in which a limited number of terminals are accessible for measurement. An efficient algorithm is described for computing network responses after a single parameter is changed. The computational complexity of this algorithm has been found to be very superior to that of a conventional algorithm.
In addition, the problem of choosing a set of measurements for fault analysis

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has been studied. We have found necessary conditions for resolving the fault to the component level. The problems of measurement error and numerical conditioning have also been studied.

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TABLE OF CONTENTS

1. The Problem of Analog Fault Analysis

Analog fault analysis is a method of finding a description (either in terms of the physical analog system or its model) of the way in which an analog system has failed. A failure (or fault) may manifest itself in many different ways, depending on the nature of the system. In the present study we consider the system to be a linear, analog electronic circuit, with at least two accessible terminals. We treat the accessible terminals as a multi-port network. The behavior of the network can be determined by measuring each of the several independent (complex) multi-port parameters (transfer functions), possibly at several frequencies. Thus, if $m/2$ independent, complex measurements are made, the network performance is given in terms of m real numbers, each of which is a function of the parameters of the network. We denote the measurement set by the real vector $\underline{\hat{M}} \overset{\Delta}{=} \text{col}\ (\hat{M}_i)$. Similarly, we denote the corresponding set of multi-port parameters computed from the network model by the real vector $\underline{M} \overset{\Delta}{=} \text{col}\ (M_i)$.

We will assume there are n real network parameters, each of which is either a resistance, capacitance, inductance, or controlled source gain. We denote the parameter set by the real vector $\underline{x} \overset{\Delta}{=} \text{col}\ (x_i)$.

The fault analysis problem may be stated as follows: given m real measurements and the network model containing n real parameters, find the values of the n real parameters which produce the m real measurements. This requires solving the system of m non-linear equations in n unknowns

$$\underline{M}\ (\underline{x}) = \underline{\hat{M}} \tag{1}$$

Currently available analog automatic test equipment (ATE) allows the preprogramming of a sequence of test signals and measurements [1]. The software interpretation of these measurements is mostly limited to a functional test of the component or module tested. To achieve greater resolution of the fault with currently available ATE (i.e., to find the faulty component or integrated circuit in the module), requires access to more terminals (test points) and the making of a more complex sequence of measurements.

Often in practice the number of terminals available for measurement is limited. For example, in troubleshooting existing equipment, available test points are fixed by the design. In the design of new equipment, it is desirable to keep the number of test points as small as possible to avoid an excessive number of pins in the connectors.

The present study was directed at improving the reliability of high resolution (down to the component or node if necessary) analog fault analysis from measurements at the accessible terminals. The measurements in this study require applying sinusoidal sources and finding the magnitude and phase relationships of the steady state responses, i.e., the measurements are a set of complex multi-port parameters at the accessible terminals. The fault is isolated by solving (1) for the n-dimensional parameter vector $\underline{x}$. Recent work in analog fault analysis has been directed toward finding an efficient way to solve these equations.

1.1 Summary of Previous Work

In solving (1), the number of network parameters may be greater than, equal to, or less than the number of measurements m. If $m < n$ it is not possible to solve for the parameter set $\underline{x}$, since in this case (1) usually has infinitely many solutions. Ransom and Saeks [2] approached this problem by finding the solution to (1) which minimizes the norm $|| \underline{x} - \underline{x}_0 ||$, where $\underline{x}_0$ is the nominal parameter set. The difficulty with this approach is that the solution assumes, roughly speaking, that the most likely state of the element values in the network is the one which causes the smallest drift from the nominal values consistant with the measurements. This assumption excludes the possibility of catastrophic faults (open and short circuits).

If $m \geq n$, then the non-linear equations given by (1) are often solved using optimization techniques by finding $\min_{\underline{x}} || \underline{M}(\underline{x}) - \hat{\underline{M}} ||$, where sometimes the norm may be reduced to zero if $m = n$. This approach was taken by Chen and Saeks [3], using an efficient algorithm to evaluate $\underline{M}(\underline{x})$. In order to accommodate catastrophic faults (open and short circuits) by this method, each element in the parameter space must be searched over the range $(0, \infty)$, which is not practical because of the prohibitively large number of evaluations of the function $\underline{M}(\underline{x})$.

In this project we continue to seek a way to overcome the difficulties of the two methods described above, viz. that catastrophic faults are overlooked altogether or that they are found at the cost of very lengthy computation. The approach described in this report is to modify the parameter set $\underline{x}$ to include the set of all single short circuits involving inaccessible nodes. The remaining parameters are branch admittances or controlled source gains. Searching over short circuits eliminates the need to search over very large values of the branch admittances. It is also unnecessary to search over very large values of controlled source gains, since this mode of failure is extremely unlikely. Johnson [4], [5] has described a very efficient algorithm for evaluating the function $\underline{M}(\underline{x})$ in the presence of a short circuit. The efficiency results from the fact that the network change caused by the short circuit results in an alteration of the (inaccessible) nodal-admittance matrix by a matrix of unit rank. As a result, the response $\underline{M}(\underline{x})$ of the faulted network can be computed (in terms of the nominal response) using many fewer multiplications than are required for an original analysis. Sparse matrix techniques were also used to reduce computation time. The efficiency of this algorithm makes it practical to exhaustively search for a minimum of $|| \underline{M}(\underline{x}) - \hat{\underline{M}} ||$ over all possible single short circuits involving an inaccessible node. The test examples described by Johnson [4], [5] show that short circuits can be reliably located by the algorithm, even though measurements were made only at one frequency, and parameter drift was not taken into account.

Performing the measurements (and analysis) at more than one frequency increases the number of measurements m without changing the number of network parameters n. This may improve the reliability of the fault analysis if all the measurements are independent. The problem of independent measurements is discussed in some detail by Sen and Saeks [6] - [8], who unfortunately stop short of describing exactly how to choose the measurements and frequencies.

2. New Results in Analog Fault Analysis

The study undertaken by the author was directed toward improving the reliability of analog fault analysis (isolating short circuits, open circuits and parameter changes) while keeping the algorithm as fast and efficient as possible. Specifically two separate studies were done:

(a) Introducing parameter changes into the current algorithm.

The current algorithm searches for a minimum of $|| \underline{M}(x) - \hat{\underline{M}} ||$ over short circuits only. By searching over the network parameters (resistances, capacitances, inductances and controlled source gains) as well, the algorithm should be improved, because of the expanded description of the fault. In addition it was hoped that the algorithm would be made more reliable (i.e., it was hoped that the algorithm would not indicate faults which were not present, but instead would find only those that were present). An efficient algorithm for calculating the network responses $\underline{M}(\underline{x})$ in the presence of a parameter change was developed and included in the search for $\min_{\underline{x}} || \underline{M}(\underline{x}) - \hat{\underline{M}} ||$. This is described in detail in Section 2.1.

(b) Independent Measurements. Increasing the dimension m of $\hat{\underline{M}}$ (and also of $\underline{M}(\underline{x})$) by adding independent measurements should improve the accuracy and resolution of the fault analysis. In most practical cases, the number of accessible terminals is fixed, so the only way to increase the number of measurements is to use more than one frequency. It is important to determine if the set of measurements is independent. The work on this problem is described in detail in Section 2.3.

We have discussed how the network parameters may be determined by minimizing the norm of the difference between the measured and computed network responses $|| \underline{M}(\underline{x}) - \hat{\underline{M}} ||$, where $\underline{M}$ and $\hat{\underline{M}}$ are both real vectors of dimension m. In the discussion which follows, the network responses are considered to be y-parameters at the accessible terminals, all referred to a common (accessible) node. Figure 1 shows a network with p + 1 accessible terminals from which we define the pxp y-parameter matrix

$$\underline{y} \triangleq \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & & & \\ y_{p1} & y_{p2} & \cdots & y_{pp} \end{bmatrix} \tag{2}$$

where, as usual,

$$y_{rs} \triangleq \left. \frac{I_s}{V_r} \right|_{V_k = 0,\ k \neq r} \tag{3}$$

In (3) I_s and V_r are the phasor response and source, respectively. The matrix $\underline{y}$ therefore represents p^2 complex

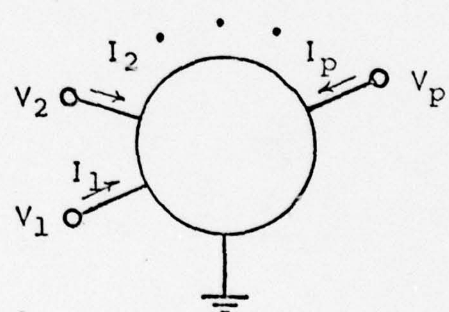

Figure 1 - p-Port Network with p+1 accessible terminals.

parameters or $2p^2$ real parameters. These $2p^2$ real parameters form the vector $\underline{M}$ of dimension m, i.e., $m \triangleq \dim \underline{M} = 2p^2$, an even integer. (The exact order in which the elements of vector $\underline{M}$ are formed from the elements of matrix $\underline{y}$ is not important. However, the measured response vector $\hat{\underline{M}}$ is formed from the measured y-parameter matrix $\hat{\underline{y}}$ by the same rule used to form one computed response vector $\underline{M}$ from the computed y-parameter matrix $\underline{y}$).

We close this section by displaying a formula derived by Johnson [4], [5], which will be used in subsequent sections.

Johnson shows that the pxp matrix of measurable y-parameters defined above is given by

$$\underline{y} = \underline{A}_1 \underline{Y}_b [\underline{1} - \underline{A}_2^T (\underline{A}_2 \underline{Y}_b \underline{A}_2^T)^{-1} \underline{A}_2 \underline{Y}_b] \underline{A}^T \tag{4}$$

where

$$\underline{A} = \left[\frac{\underline{A}_1}{\underline{A}_2} \right]$$

is the *node-to-branch incidence matrix* (as defined by and Kuh [9] partitioned into the accessible ($\underline{A}_1$) and the inaccessible ($\underline{A}_2$) nodes, and $\underline{Y}_b$ is the branch admittance matrix [4], [5], [10]. If the network contains b branches, $\underline{Y}_b$ is a sparse bxb matrix whose elements are simple functions of the branch admittances and the controlled source gains. It therefore plays the role of the parameter vector $\underline{x}$ and can be expressed in terms of $\underline{x}$.

2.1 Parameter Changes

In order to find $\min_{\underline{Y}_b} || \underline{y}(\underline{Y}_b) - \hat{\underline{y}} ||$ it is important to compute $\underline{y}(\underline{Y}_b)$ efficiently. Such an algorithm has been found if a change is made in only one parameter at a time. To derive this result, suppose $\underline{Y}_b$ is changed by the bxb matrix $\underline{\Delta}$. Then from (4) the y-parameter matrix becomes

$$\underline{y}' = \underline{A}_1 (\underline{Y}_b+\underline{\Delta}) [\underline{1} - \underline{A}_2^T (\underline{A}_2(\underline{Y}_b+\underline{\Delta}) \underline{A}_2^T)^{-1} \underline{A}_2(\underline{Y}_b+\underline{\Delta})] \underline{A}^T \tag{5}$$

The major computational difficulty in evaluating the right side of (5) is computing the inverse of the new inaccessible nodal admittance matrix $\underline{Y}_2' = \underline{A}_2 (\underline{Y}_b+\underline{\Delta}) \underline{A}_2^T$. Define the qxq matrix $\underline{Y}_2 \triangleq \underline{A}_2\underline{Y}_b\underline{A}_2^T$, where q is the number of inacessible nodes. Thus $\underline{Y}_2' = \underline{Y}_2 + \underline{A}_2\underline{\Delta}\ \underline{A}_2^T$ and we wish to find the inverse of $\underline{Y}_2'$.

During this investigation we have discovered that if only one network parameter is changed, $\underline{\Delta}$ can be formed as the outer product of two vectors $\underline{u} \triangleq \text{col}\ (u_1)$ and $v \triangleq \text{col}\ (v_i)$, i.e. $\underline{\Delta} = \underline{uv}^T$. (In this case $\underline{\Delta}$ has unit rank). The specific way in which $\underline{u}$ and $\underline{v}$ are formed depend on whether the network parameter changed is a passive admittance or a controlled source. The several special cases are discussed in Sections 2.1.1 through 2.1.5, which the reader may skip without loss of continuity.

The product $\underline{A}_2\underline{\Delta}\ \underline{A}_2^T$ may therefore be expressed as the outer product $\underline{UV}^T$, where $\underline{U} \triangleq \underline{A}_2\underline{u}$ and $\underline{V} \triangleq \underline{A}_2\underline{v}$ are both vectors of dimension q, where q is the number of inaccessible nodes. We may now write

$$\underline{Y}_2' = \underline{Y}_2 + \underline{UV}^T. \tag{6}$$

Householder [11] shows that the inverse of $\underline{Y}_2'$ can be formed from

$$\underline{Y}_2'^{-1} = \underline{Y}_2^{-1} [\underline{1} - \underline{U}(1 + \underline{V}^T\underline{Y}_2^{-1}\underline{U})^{-1} \underline{V}^T \underline{Y}_2^{-1}] \tag{7}$$

In (7) the inverse of $\underline{Y}_2$ is known from an initial computation (in terms of the nominal parameter values) which is performed only once, and $1 + \underline{V}^T \underline{Y}_2^{-1} \underline{U}$ is a scaler. Thus the right side of (7) may be found without a matrix inversion.

The Fortran code for computing $\underline{Y}_2^{-1}$ using (7) has been written and may be found in Appendix 1 (SUBROUTINE Y2PI2). Sparse matrix techniques have been used to store the vectors $\underline{U}$ and $\underline{V}$.

The purpose of the next several sections (2.1.1 through 2.1.5 is to show that a change in any network parameter changes the branch admittance $\underline{Y}_b$ by a matrix of unit rank, i.e., $\underline{Y}_b' = \underline{Y}_b + \underline{\Delta}$, where $\underline{\Delta} = \underline{u}\,\underline{v}^T$, and $\underline{u} \triangleq \text{col}\,(u_i)$, $\underline{v} \triangleq \text{col}\,(v_i)$. An efficient computer program for forming the vectors $\underline{u}$ and $\underline{v}$ using sparse matrix techniques has been written, and the listing may be found in Appendix 1 (SUBROUTINE UVVEC).

Johnson and Pennington [10] have shown that in a network containing b branches, the bxb branch admittance matrix can be formed from the element matrices as follows:

$$\underline{Y}_b = (\underline{1} + \underline{Y}\,\underline{R}_m - \underline{\alpha})^{-1}\,(\underline{Y} - \underline{Y}\,\underline{M} + \underline{G}_m) \tag{8}$$

where $\underline{1}$ is the bxb identity matrix, $\underline{Y}$ is the bxb matrix diag (Y_i) and Y_i is the self-admittance of branch i; $\underline{R}_m$ is the bxb matrix $\{R_{ij}\}$, where R_{ij} is the transistance in branch j controlled by the current in branch i; $\underline{\alpha}$, $\underline{\mu}$, and $\underline{G}_m$ are defined similarly and contain branch coupling in the form of current gains, voltage gains, and transconductances respectively.

2.1.1 Parameter Change is a Passive Admittance†

If a change is made in branch admittance k, then $\underline{Y}$ becomes $\underline{Y} + \underline{\Delta}$, where $\underline{\Delta}$ is zero except for element k. A little algebra shows that $\underline{Y}_b$ has been changed to $\underline{Y}_b' = \underline{Y}_b + \underline{\Delta}$, where

†The reader may omit this section without loss of continuity.

$$\underline{\Delta} = \underline{W}\,(\Delta\underline{X}) - (\Delta\underline{W})\,\underline{X} - (\Delta\underline{W})\,(\Delta\underline{X}) \tag{9}$$

and

$$\underline{X} = \underline{Y} - \underline{Y}\,\underline{\mu} + \underline{G}_m$$

$$\underline{W} = \underline{1} - \underline{Y}\,\underline{R}_m + \underline{\alpha}$$

$$\Delta\underline{X} = \underline{\Delta}_y - \underline{\Delta}_y\,\underline{\mu}$$

$$\Delta\underline{W} = -\Delta_y\,\underline{R}_m$$

Equation (9) simplifies since $(\Delta\underline{W})\,(\Delta\underline{X}) \equiv \underline{o}$, which may be shown as follows. After some algebra, using (9),

$$(\Delta\underline{W})\,(\Delta\underline{X}) = (\underline{\Delta}_y\underline{R}_m\,\underline{\Delta}_y)\,(\underline{\mu} - \underline{1}),$$

which is zero if $\underline{\Delta}_y\,\underline{R}_m\,\underline{\Delta}_y = \underline{0}$. Suppose branch admittance k is changed, and define $\underline{B} \triangleq \underline{R}_m\,\underline{\Delta}_y$. Therefore $B_{ij} = \sum_m (R_m)_{im}\,(\Delta_y)_{mj}$. But all elements of $\underline{\Delta}_y$ are zero except for element (k,k).

$$\begin{aligned} B_{ij} &= 0 \ , \ j \neq k \\ B_{ik} &= (R_m)_{ik}\,(\Delta_y)_{kk} \quad , \quad \forall\, i \end{aligned} \tag{10}$$

Now $\underline{\Delta}_y\underline{R}_m\,\underline{\Delta}_y = \underline{\Delta}_y\,\underline{B}$. Consider the elements of column k of this matrix:

$$(\underline{\Delta}_y\,\underline{R}_m\,\underline{\Delta}_y)_{rk} = \sum_i (\Delta_y)_{ri}\,B_{ik} \tag{11}$$

From (10) we see that $\underline{\Delta}_y\,\underline{R}_m\,\underline{\Delta}_y$ is zero except for elements in column k. Furthermore the summation need be carried out over element i = r only, since $(\Delta_y)_{ri} = 0$, $r \neq i$. Finally, $(\Delta_y)_{rr} = 0$ unless r = k, so $\underline{\Delta}_y\,\underline{R}_m\,\underline{\Delta}_y$ contains only one non-zero element, namely $(\underline{\Delta}_y\,\underline{R}_m\,\underline{\Delta}_y)_{kk} = (\Delta_y)_{kk}(R_m)_{kk}(\Delta_y)_{kk}$. But this last expression is zero because branch k never contains a trans-resistance controlled by branch k. (This is modelled instead as a simple self admittance). QED.

After deleting the last term in (9), the change in the branch admittance matrix $\underline{Y}$ due to a change in branch admittance k becomes

$$\underline{\Delta} = \underline{\Delta}_y - \underline{\Delta}_y \underline{\mu} + \hat{\underline{\alpha}} \underline{\Delta}_y - \hat{\underline{\alpha}} \underline{\Delta}_y \underline{\mu} + \underline{\Delta}_y \underline{R}_m \underline{Y} + \underline{\Delta}_y \underline{R}_m \hat{\underline{G}}_m \tag{12}$$

where $\hat{\underline{\alpha}} \triangleq \underline{\alpha} - \underline{Y}\,\underline{R}_m$ is the Norton equivalent current coupling matrix and $\hat{\underline{G}}_m \triangleq \underline{G}_m - \underline{Y}\,\underline{\mu}$ is the Norton equivalent transconductance matrix (see Johnson and Pennington [10]). We will now show that all terms on the right side of (12) contribute elements only to one row or column (except the term $\hat{\underline{\alpha}}\,\underline{\Delta}_y$, q.v.), if only one network parameter is changed. As a result, $\underline{\Delta}$ may be written $\underline{u}\,\underline{v}^T$, as explained in Section 2.1.

1. $\underline{\Delta}_y$ places a term in element (k,k) of $\underline{\Delta}$.
2. $$(\underline{\Delta}_y \underline{\mu})_{ij} = \sum_m (\Delta_y)_{im} \mu_{mj} = (\Delta_y)_{ii}\, \mu_{ij} = 0\ ,\ i \neq k$$

Therefore $\underline{\Delta}_y \underline{\mu}$ places elements only in row k of $\underline{\Delta}$.

3. $$(\hat{\underline{\alpha}} \underline{\Delta}_y)_{ij} = \sum_m \hat{\alpha}_{im} (\Delta_y)_{mj} = \hat{\alpha}_{ij} (\Delta_y)_{jj} = 0\ ,\ j \neq k$$

Therefore $\hat{\underline{\alpha}}\,\underline{\Delta}_y$ places elements in column k of $\underline{\Delta}$, if branch k contains current controlled sources.

4. $$(\underline{\alpha}\, \underline{\Delta}_y \underline{\mu})_{ij} = \sum_m \hat{\alpha}_{im} (\underline{\Delta}_y \underline{\mu})_{mj} = \hat{\alpha}_{iK} (\underline{\Delta}_y \underline{\mu})_{Kj} = \hat{\alpha}_{iK} (\Delta_y)_{KK}\, \mu_{Kj}$$

If such a term were present, the network model would contain a controlled source in branch i ($\hat{\alpha}_{ik}$) which was controlled by the branch current of a branch containing a controlled source (branch k contains source μ_{kj}). Such a condition is not allowed in the model [10], so $\underline{\alpha}\ \underline{\Delta}_y\ \underline{\mu} = \underline{0}$.

The final two terms in (12) contain the product $\underline{\Delta}_y\ \underline{R}_m$. Element (i, j) of this matrix is

$$(\underline{\Delta}_y\ \underline{R}_m)_{ij} = \sum_m (\Delta_y)_{im}\ (R_m)_{mj}$$

$$= (\Delta_y)_{ii}\ (R_m)_{ij} = 0\ ,\ i \neq k$$

$$(\underline{\Delta}_y\ \underline{R}_m)_{ki} = (\Delta_y)_{kk}\ (R_m)_{kj}\ ,\ i = k$$

We now consider the final two terms in (12).

5. $$(\underline{\Delta}_y\ \underline{R}_m\ \underline{Y})_{ij} = \sum_m (\underline{\Delta}_y\ \underline{R}_m)_{im}\ Y_{mj}$$

$$= (\underline{\Delta}_y\ \underline{R}_m)_{ij}\ Y_{jj} = 0\ ,\ i \neq k$$

$$(\underline{\Delta}_y\ \underline{R}_m\ \underline{Y})_{kj} = (\Delta_y)_{kk}\ (R_m)_{kj}\ Y_{ij} \qquad \forall\ j$$

Therefore $\underline{\Delta}_y\ \underline{R}_m\ \underline{Y}$ contributes terms along row k of $\underline{\Delta}$.

6. $$(\underline{\Delta}_y\ \underline{R}_m\ \underline{\hat{G}}_m)_{ij} = \sum_m (\underline{\Delta}_y\ \underline{R}_m)_{im}\ (\hat{G}_m)_{mj} = 0\ ,\ i \neq k$$

$$(\underline{\Delta}_y\ \underline{R}_m\ \underline{\hat{G}}_m)_{\ j} = \sum_m (\underline{\Delta}_y\ \underline{R}_m)_{km}\ (\hat{G}_m)_{mj}$$

$$= \sum_m (\Delta_y)_{kk}\ (R_m)_{km}\ (\hat{G}_m)_{mj}$$

$$= (\Delta_y)_{kk} \sum_m (R_m)_{km}\ (\hat{G}_m)_{mj} \qquad \forall\ j$$

Therefore $\underline{\Delta}_y\ \underline{R}_m\ \underline{\hat{G}}_m$ contributes terms along row k of $\underline{\Delta}$, although few, if any, such terms are likely to exist since the presence of this term requires that the network contain a transresistance controlled by the current in a branch containing a voltage controlled source.

2.1.2 Parameter Change is a Transresistance[†]

If a change is made in transresistance $(R_m)_{ij}$, then $\underline{R}_m$ becomes $\underline{R}_m + \underline{\Delta}_r$, where $\underline{\Delta}_r$ is zero except for element $(\Delta_r)_{ij}$. Replacing $\underline{R}_m$ by $\underline{R}_m + \underline{\Delta}_r$ in (8) and performing some algebraic reduction, $\underline{Y}_b$ becomes $\underline{Y}_b + \underline{\Delta}$ where

$$\underline{\Delta} = - \underline{Y}\,\underline{\Delta}_r\,\underline{Y} - \underline{Y}\,\underline{\Delta}_r\,\hat{\underline{G}}_m \qquad (13)$$

Both terms in (13) contain the product $\underline{Y}\,\underline{\Delta}_r$. Expanding element (r, s) of $\underline{Y}\,\underline{\Delta}_r$

$$(\underline{Y}\,\underline{\Delta}_r)_{rs} = \sum_m Y_{rm}\,(\Delta_r)_{ms}$$

$$= Y_{rr}\,(\Delta_r)_{rs} = 0 \quad \text{if } r \neq i, \; s \neq j$$

$$(\underline{Y}\,\underline{\Delta}_r)_{ij} = Y_{ii}\,(\Delta_r)_{ij}$$

Therefore $\underline{Y}\,\underline{\Delta}_r$ has only one non-zero element (i, j). We now expand element (r, s) of both terms of (13).

1. $$(\underline{Y}\,\underline{\Delta}_r\,\underline{Y})_{rs} = \sum_m (\underline{Y}\,\underline{\Delta}_r)_{rm}\,Y_{ms}$$

$$= (\underline{Y}\,\underline{\Delta}_r)_{rs}\,Y_{ss}$$

$$= 0\,, \; r \neq i\,, \; s \neq j$$

$$(\underline{Y}\,\underline{\Delta}_r\,\underline{Y})_{ij} = Y_{ii}\,(\Delta_r)_{ij}\,Y_{jj}$$

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† The reader may skip this section without loss of continuity.

Therefore $\underline{Y}\ \underline{\Delta}_r\ \underline{Y}$ contains a single non-zero element (i, j).

2. $$(\underline{Y}\ \underline{\Delta}_r\ \underline{\hat{G}}_m)_{rs} = \sum_m (\underline{Y}\ \underline{\Delta}_r)_{rm}\ (\hat{G}_m)_{ms}$$
$$= 0\ ,\ r \neq i,\ m \neq j$$

$$(\underline{Y}\ \underline{\Delta}_r\ \underline{\hat{G}}_m)_{is} = (\underline{Y}\ \underline{\Delta}_r)_{ij}\ (\hat{G}_m)_{js}$$
$$= Y_{ii}\ (\Delta_r)_{ij}\ (\hat{G}_m)_{js}\ ,\ \forall\ s$$

Therefore $\underline{Y}\ \underline{\Delta}_r\ \underline{G}_m$ places terms in row i of $\underline{\Delta}$.

2.1.3 Parameter Change is a Current Gain†

If a change is made in current gain α_{ij} , then $\underline{\alpha}$ is changed to $\underline{\alpha} + \underline{\Delta}_\alpha$, where $\underline{\Delta}_\alpha$ is zero except for element $(\Delta_\alpha)_{ij}$. Replacing $\underline{\alpha}$ by $\underline{\alpha} + \underline{\Delta}_\alpha$ in (8), after some algebraic reduction $\underline{Y}_b$ becomes $\underline{Y}_b + \underline{\Delta}$, where

$$\underline{\Delta} = -\ \underline{\Delta}_\alpha\ \underline{Y} - \underline{\Delta}_\alpha\ \underline{\hat{G}}_m \tag{14}$$

We now expand element (r, s) of both terms of (14).

1. $$(\underline{\Delta}_\alpha\ \underline{Y})_{rs} = \sum_m (\Delta_\alpha)_{rm}\ Y_{ms}$$
$$= (\Delta_\alpha)_{rs}\ Y_{ss} = 0\ ,\ r \neq i,\ s \neq j$$
$$(\underline{\Delta}_\alpha\ \underline{Y})_{ij} = (\Delta_\alpha)_{ij}\ Y_{jj}$$

Therefore $\underline{\Delta}_\alpha\ \underline{Y}$ contains a single non-zero element (i, j) .

2. $$(\underline{\Delta}_\alpha\ \underline{\hat{G}}_m)_{rs} = \sum_m (\Delta_\alpha)_{rm}\ (\hat{G}_m)_{ms}$$
$$= 0\ ,\ r \neq i,\ m \neq j$$
$$(\underline{\Delta}_\alpha\ \underline{G}_m)_{is} = (\Delta_\alpha)_{ij}\ (\hat{G}_m)_{js}\ ,\ \forall\ s$$

Therefore $\underline{\Delta}_\alpha\ \underline{G}_m$ contributes elements in row i of $\underline{\Delta}$.

---

† The reader may skip this section without loss of continuity.

2.1.4 Parameter Change is a Voltage Gain†

If a change is made in voltage gain μ_{ij} , then $\underline{\mu}$ is changed to $\underline{\mu} + \underline{\Delta}_\mu$, where $\underline{\Delta}_\mu$ is zero except for element $(\Delta_\mu)_{ij}$. Replacing $\underline{\mu}$ by $\underline{\mu} + \underline{\Delta}_\mu$ in (8), after some algebraic reduction $\underline{Y}_b$ becomes $\underline{Y}_b + \underline{\Delta}$, where

$$\underline{\Delta} = - \underline{Y}\,\underline{\Delta}_\mu - \underline{\hat{\alpha}}\,\underline{Y}\,\underline{\Delta}_\mu \tag{15}$$

We now expand element (r, s) of both terms of (15).

1. $$(\underline{Y}\,\underline{\Delta}_\mu)_{rs} = \sum_m Y_{rm}\,(\Delta_\mu)_{ms}$$

$$= Y_{rr}\,(\Delta_\mu)_{rs} = 0 \;,\; r \neq i \;,\; s \neq j$$

$$(\underline{Y}\,\underline{\Delta}_\mu)_{ij} = Y_{ii}\,(\Delta_\mu)_{ij}$$

Therefore $\underline{Y}\,\underline{\Delta}_\mu$ contributes only one term to $\underline{\Delta}$.

2. $$(\underline{\hat{\alpha}}\,\underline{Y}\,\underline{\Delta}_\mu)_{rs} = \sum_m \hat{\alpha}_{rm}\,(\underline{Y}\,\underline{\Delta}_\mu)_{ms}$$

$$= 0 \;,\; m \neq i \;,\; s \neq j$$

$$(\underline{\hat{\alpha}}\,\underline{Y}\,\underline{\Delta}_\mu)_{rj} = \hat{\alpha}_{ri}\,(\underline{Y}\,\underline{\Delta}_\mu)_{ij}$$

$$= \hat{\alpha}_{ri}\,Y_{ii}\,(\Delta_\mu)_{ij} \;,\; \forall\, r$$

Therefore $\underline{\hat{\alpha}}\,\underline{Y}\,\underline{\Delta}_\mu$ contributes terms down column j of $\underline{\Delta}$.

2.1.5 Parameter Change is a Transconductance†

If a change is made in transconductance $(G_m)_{ij}$ then $\underline{G}_m$ is changed to $\underline{G}_m + \underline{\Delta}_g$, where $\underline{\Delta}_g$ is zero except for element $(\underline{\Delta}_g)_{ij}$.

---

† The reader may skip this section without loss of continuity.

Replacing $\underline{G}_m$ by $\underline{G}_m + \underline{\Delta}_g$ in (8), after some algebraic reduction $\underline{Y}_b$ becomes $\underline{Y}_b + \underline{\Delta}$, where

$$\underline{\Delta} = \underline{\Delta}_g + \underline{\hat{\alpha}}\ \underline{\Delta}_g \tag{16}$$

We now discuss both terms in (16).

1. $\underline{\Delta}_g$ contributes only one term, $(\Delta_g)_{ij}$, to $\underline{\Delta}$.
2. $(\underline{\hat{\alpha}}\ \underline{\Delta}_g)_{rs} = \sum_m \hat{\alpha}_{rm} (\Delta_g)_{ms} = 0$, $m \neq i$, $s \neq j$

   $(\underline{\hat{\alpha}}\ \underline{\Delta}_g)_{rj} = \hat{\alpha}_{ri} (\Delta_g)_{ij}$, $\forall$ r

Therefore $\underline{\hat{\alpha}}\ \underline{\Delta}_g$ contribute terms down column j of $\underline{\Delta}$.

2.2 Finding Independent Measurements

In Section 1 we discussed the problem of solving the nonlinear equation $\underline{M}(\underline{x}) = \hat{\underline{M}}$, in which $\underline{x}$ is the unknown parameter vector of dimension n, $\hat{\underline{M}}$ is the vector of m network responses measured at the accessible terminals, and $\underline{M}\ (\underline{x})$ is the vector of m network responses calculated from the network model. The three cases (m<n, m=n, m>n) must be considered separately. Ransom and Saeks [2] have discussed the underdetermined case (m<n), which is not very practical because the fault cannot be resolved satisfactorily in this case. The second and third cases ($m \geq n$) can be solved by finding the parameter vector $\underline{x}$ which minimizes the norm $||\underline{M}(\underline{x}) - \hat{\underline{M}}||$. (This approach has the advantage of taking measurement error into account in a sensible way in the overdetermined case, m>n). In Section 2, we discussed the solution to this optimization problem by means of a search over the parameter set $\underline{x}$. An alternative approach, which provides some insight into the question of independent measurements, is to expand $\underline{M}(\underline{x})$ in a Taylor series about the nominal parameter set $\underline{x}_o$. If we truncate the Taylor series after the linear terms, we obtain the approximation

$$\underline{M}(\underline{x}) \approx \underline{M}(\underline{x}_o) + \underline{F}(\underline{x}_o)(\underline{x} - \underline{x}_o) \tag{16}$$

where $\underline{F}(\underline{x})$ is the Jacobian Matrix defined by

$$\left\{\underline{F}(\underline{x})\right\}_{ij} \triangleq \frac{\partial\{\underline{M}(\underline{x})\}_i}{\partial x_j} \qquad \left\{\begin{array}{l} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, n \end{array}\right\} \tag{17}$$

From (16) the linearized form of the equation $\underline{M}(\underline{x}) = \hat{\underline{M}}$ is determined to be $\underline{F}(\underline{x}_o)(\underline{x} - \underline{x}_o) = \hat{\underline{M}} - \underline{M}(\underline{x}_o)$, in which, as before, $\dim(\underline{x} - \underline{x}_o) = n$ and $\dim(\hat{\underline{M}}-\underline{M}) = m$. Since we are treating the overdetermined case m>n, we proceed by minimizing $||\underline{F}_o(\underline{x}-\underline{x}_o) - (\hat{\underline{M}}-\underline{M}_o)||$, where $\underline{F}_o \triangleq \underline{F}(\underline{x}_o)$ and $\underline{M}_o \triangleq \underline{M}(\underline{x}_o)$. The minimization of the norm is accomplished by first defining the scalar function

$$
\begin{aligned}
\phi(\underline{x}) &\triangleq ||\underline{F}_o(\underline{x}-\underline{x}_o) - (\underline{M}-\underline{M}_o)||^2 \\
&= [\underline{F}_o(\underline{x}-\underline{x}_o) - (\underline{M}-\underline{M}_o)]^T [\underline{F}_o(\underline{x}-\underline{x}_o) - (\underline{M}-\underline{M}_o)] \\
&= (\underline{x}-\underline{x}_o)^T \underline{F}_o^T \underline{F}_o (\underline{x}-\underline{x}_o) \\
&\quad -2(\underline{M}-\underline{M}_o)^T \underline{F}_o (\underline{x}-\underline{x}_o) \\
&\quad + (\underline{M}-\underline{M}_o)^T (\underline{M}-\underline{M}_o)
\end{aligned}
\tag{18}
$$

and then taking the differential of $\phi(\underline{x})$:

$$
\begin{aligned}
d\phi = 2[(\underline{x}-\underline{x}_o)^T \underline{F}_o^T \underline{F}_o - (\underline{M}-\underline{M}_o)^T \underline{F}_o]d\underline{x} \\
+ d\underline{x}^T \underline{F}_o^T \underline{F}_o d\underline{x}
\end{aligned}
\tag{19}
$$

At a minimum of $\phi(\underline{x})$, $d\phi$ may not change sign when $d\underline{x}$ changes sign. Therefore a necessary condition for a minimum of $\phi(\underline{x})$ is

$$(\underline{x}-\underline{x}_o)^T \underline{F}_o^T\underline{F}_o - (\underline{M}-\underline{M}_o)^T \underline{F}_o = \underline{0}$$

or

$$\underline{F}_o^T\underline{F}_o (\underline{x}-\underline{x}_o) = \underline{F}_o^T (\underline{M}-\underline{M}_o) . \tag{20}$$

The deviation $\underline{x}-\underline{x}_o$ of the element values from their nominal value may be found from (20) if the nxn matrix $\underline{F}_o^T\underline{F}$ is non-singular, which requires in this case that rank $(\underline{F}) = n$.[†]

[†] We do not necessarily recommend using (20) to determine the parameter set $\underline{x}$ of the faulted network, although this may be the most efficient approach. See Section 3 for a discussion of computational complexity.

Since $m \geq n$, the mxn Jacobian matrix $\underline{F}$ has at least as many rows as columns, and the maximum rank of $\underline{F}$ is n. The rank of $\underline{F}$ is less than n if any column of $\underline{F}$ is a linear combination of the remaining columns. The simplest case is that a column is a constant multiple of another column. Suppose, for example, (col i) = a (col j), i.e.

$$\frac{\partial M_k}{\partial x_i} = a \frac{\partial M_k}{\partial x_j} \quad (k = 1, 2, \ldots, m) \tag{21}$$

One obvious way this can occur is if parameters x_i and x_j are two like elements in series or parallel. Suppose that x_i and x_j are the conductances of two resistors in parallel, so that $y \triangleq x_i + x_j$ is the combined conductance. All transfer functions which do not access separately the inputs or outputs of x_i and x_j may be written $M_k(y, \underline{z})$, where $\underline{z}$ is the vector of all network parameters except x_i and x_j. Now the partial derivatives of M_k with respect to x_i and x_j are

$$\frac{\partial M_k}{\partial x_i} = \frac{\partial M_k}{\partial y} \cdot \frac{\partial y}{\partial x_i} = \frac{\partial M_k}{\partial y} \tag{22}$$

$$\frac{\partial M_k}{\partial x_i} = \frac{\partial M_k}{\partial y} \cdot \frac{\partial y}{\partial x_i} = \frac{\partial M_k}{\partial y} \tag{23}$$

Thus (21) applies with a=1, as asserted. This result, that no distinction can be made between imbedded like element kinds in series or parallel, is intuitively reasonable.

A less obvious example of column dependency is illustrated by the simplified cascaded amplifier circuit shown in Figure 2. Define the transfer functions and network parameters

$$M_1 = z_{11} = R_1 \qquad x_1 = R_1$$

$$M_2 = z_{21} = g_{m1}g_{m2}R_1R_2R_3 \qquad x_2 = R_2$$

$$M_3 = z_{22} = R_3 \qquad x_3 = R_3$$

$$x_4 = g_{m1}$$

$$x_5 = g_{m2}$$

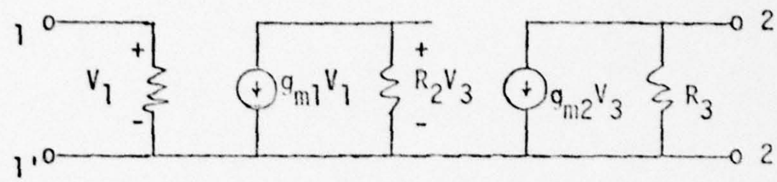

Figure 2 - Simplified Cascaded Amplifier

The transimpedance z_{12} is not used in this example because its value is zero. The Jacobian matrix $\left\{\partial M_i/\partial x_j\right\}$ is

$$\underline{F} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ g_{m1}g_{m2}R_2R_3 & g_{m1}g_{m2}R_1R_3 & g_{m1}g_{m2}R_1R_2 & g_{m2}R_1R_2R_3 & g_{m1}R_1R_2R_3 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If g_{m1} and g_{m2} are equal, then columns 4 and 5 are identical. This doesn't reduce the rank of $\underline{F}$ in this case (the rank of $\underline{F}$ is 3), but it could in practical cases where $\underline{F}$ has more rows than columns (m>n).

Continuing with the problem of column dependency, suppose x_i is the conductance of a resistor in parallel with a capacitor of capacitance x_j, and $x = x_i + j\omega x_j$ is the combined admittance. All transfer functions which do not access separately the inputs or outputs of parameters x_i and x_j may be written $M_K(x,\underline{z})$, where $\underline{z}$ is the vector of all network parameters except x_i and x_j. The partial derivatives of M_K with respect to x_i and x_j are

$$\frac{\partial M_k}{\partial x_i} = \frac{\partial M_k}{\partial x} \cdot \frac{\partial x}{\partial x_i} = \frac{\partial M_K}{\partial x} \tag{24}$$

$$\frac{\partial M_k}{\partial x_j} = \frac{\partial M_k}{\partial x} \cdot \frac{\partial x}{\partial x_j} = j\omega \frac{\partial M_k}{\partial x} \tag{25}$$

Thus (21) applies with $a=j\omega$. Columns i and j of $\underline{F}$ are dependent, so rank $\underline{F} < n$ and the nxn matrix $\underline{F}^T\underline{F}$ cannot be inverted, since rank $\underline{F}^T\underline{F} < n$, if $m \geq n$. As a consequence, (20) cannot be solved for the new parameter vector. We have discovered the surprising result that the identification problem cannot be perfectly resolved if the number of measurements is at least as great as the number of parameters and the network contains imbedded elements in parallel (or in series). That rank $(\underline{F}) < n$ in this case will be illustrated now by an example.

In this example it is necessary that $m \geq n$, and we have chosen the simple 2-port network illustrated in Figure 3 in which m=n=3.

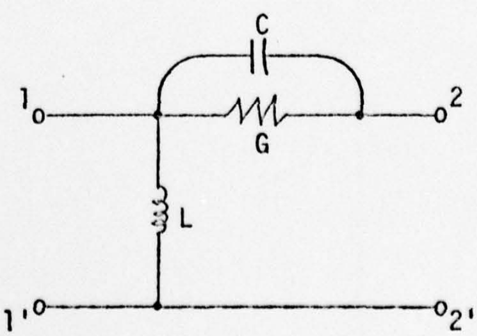

Figure 3 - Two-port Network.

We choose the three transfer functions and parameters

$$M_1 = y_{11} = \frac{1}{j\omega L} + j\omega C + G \qquad x_1 = L$$

$$M_2 = y_{21} = j\omega C + G \qquad x_2 = C$$

$$M_3 = y_{22} = j\omega C + G \qquad x_3 = G$$

The Jacobian matrix $\left\{ \frac{\partial M_i}{\partial K_j} \right\}$ is

$$\underline{F} = \begin{bmatrix} -\frac{1}{j\omega L^2} & j\omega & 1 \\ 0 & j\omega & 1 \\ 0 & j\omega & 1 \end{bmatrix}$$

as asserted, columns 2 and 3 of $\underline{F}$ are dependent. Since rank $\underline{F} = 2$, the fault analysis problem cannot be fully resolved for this problem.

In some cases the column dependency can be removed by increasing the number of measurements by using another frequency. In this example if M_1, M_2 and M_3 are defined as before at frequency $\omega=\omega_1$ and M_4, M_5 and M_6 are the same transfer functions at frequency $\omega=\omega_2$, the Jacobian matrix is

$$\underline{F} = \begin{bmatrix} -\frac{1}{j\omega_1 L^2} & j\omega_1 & 1 \\ 0 & j\omega_1 & 1 \\ 0 & j\omega_1 & 1 \\ -\frac{1}{j\omega_2 L^2} & j\omega_2 & 1 \\ 0 & j\omega_2 & 1 \\ 0 & j\omega_2 & 1 \end{bmatrix}$$

and rank $(\underline{F}) = 3$.

Another way to handle the problem is to split the transfer functions into their real and imaginary parts. Continuing the example, define the transfer functions and parameters

$$
\begin{aligned}
M_1 &\triangleq Re(y_{11}) = G & x_1 &= L\\
M_2 &\triangleq Im(y_{11}) = \omega C - \frac{1}{\omega L} & x_2 &= C\\
M_3 &\triangleq Re(y_{21}) = G & x_3 &= G\\
M_4 &\triangleq Im(y_{21}) = \omega C & &
\end{aligned}
$$

Now $m = 4$, $n = 3$, and the Jacobian matrix is

$$
\underline{F} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{\omega L^2} & \omega & 0 \\ 0 & 0 & 1 \\ 0 & \omega & 0 \end{bmatrix}
$$

and again rank $(\underline{F}) = 3$. Since M_1 and M_3 are identical, rows 1 and 3 of $\underline{F}$ are the same. This doesn't reduce the rank of $\underline{F}$, but it is clear that M_3 may be discarded without reducing the information of the measurements.

From the discussion above we conclude that the measurements should be split into their real and imaginary parts, and more than one frequency should be used, if necessary, to make the number of measurements at least as great as the number of network parameters ($m \geq n$).

In practical examples a considerable amount of computation is needed to find the functional form of the elements of the Jacobian matrix. Sen and Saeks use such functional forms in their discussion of the problem of independent measurements [6] - [8], but admit that no theory yet exists for choosing measurements while taking numerical considerations into account.

In view of the discussion above, it appears that the most practical approach to the problem of identifying a sufficient number of independent measurements ($m \geq n$) is to examine the numerical conditioning of the nxn matrix $\underline{F}^T\underline{F}$, evaluated from the nominal network parameters. An algorithm for evaluating cond $(\underline{F}^T F)^\dagger$ is given by Forsythe and Moler [12].

Establishing a useful set of independent measurements requires searching for min cond $(\underline{F}^T\underline{F})$ over various sets of trial measurements and frequencies. This time consuming search seems most suitable to a "simulation before test" process; i.e., the best set of measurements is established from the circuit model before testing the faulty circuit. The contract period was not sufficiently long to permit trying this procedure, but a working algorithm is in preparation.

2.2.1 Computing the Jacobian Matrix

The Jacobian matrix is defined by $\underline{F} \triangleq [\partial M_i(\underline{x})/\partial x_j]$. We have taken the network responses $\underline{M}(\underline{x})$ to be the real and imaginary parts of the y-parameters at the accessible terminals, where, from (4),

$$\underline{y} = \underline{A}_1\underline{Y}_b\underline{A}_1^T - \underline{A}_1\underline{Y}_b\underline{A}_2^T\,(\underline{A}_2\underline{Y}_b\underline{A}_2^T)^{-1}\,\underline{A}_2\underline{Y}_b\underline{A}_1^T \tag{26}$$

$$\underline{Y}_b = (\underline{1} - \underline{Y}\,\underline{R}_m + \underline{\alpha})\,(\underline{Y} - \underline{Y}\,\underline{M} + \underline{G}_m)$$

† If $\underline{A}$ is a square matrix, cond $(\underline{A})$ is defined to be the ratio of the largest to smallest eigenvalue in absolute value. Therefore cond $(\underline{A}) \geq 1$, and large values for cond $(\underline{A})$ indicate nearly singular matrices for which the numerical evaluation of $\underline{A}^{-1}$ may be meaningless.

and so we want to compute $\partial \underline{y}/\partial x_i$. Making use of the identity

$$\frac{\partial \underline{B}^{-1}}{\partial x} = - \underline{B}^{-1} \frac{\partial \underline{B}}{\partial x} \underline{B}^{-1}$$

and after much algebraic simplification, we find that

$$\frac{\partial \underline{y}}{\partial x_i} = \underline{A}_1 \left\{ [\underline{1} - \underline{Y}_b \underline{A}_2^T (\underline{A}_2 \underline{Y}_b \underline{A}^T)^{-1} \underline{A}_2] \frac{\partial \underline{Y}_b}{\partial x_i} [\underline{1} - \underline{A}_2^T (\underline{A}_2 \underline{Y}_b \underline{A}_2^T)^{-1} \underline{A}_2 \underline{Y}_b] \right\} \underline{A}_1^T \tag{27}$$

In (27) the second term in square brackets, $[\underline{1} - \underline{A}_2^T (\underline{A}_2 \underline{Y}_b \underline{A}_2^T)^{-1} \underline{A}_2 \underline{Y}_b]$, is available from the nominal analysis.† The first term in square brackets, $[\underline{1} - \underline{Y}_b \underline{A}_2^T (\underline{A}_2 \underline{Y}_b \underline{A}^T)^{-1} \underline{A}_2]$, is very similar to the second term in square brackets. In fact, if the network model contained no controlled sources, $\underline{Y}_b$ would be symmetrical, and these two terms would be the transpose of one another. The right side of (27) could be computed very efficiently in this case, especially since $\partial \underline{Y}_b/\partial x_i$ is very sparse. Unfortunately we cannot take advantage of this simplification, because the circuits of greatest concern contain controlled sources.

An algorithm for computing the right side of (27) as efficiently as possible, taking advantage of the sparseness of $\partial \underline{Y}_b/\partial x_i$, has been written. The reader is referred to the listing of subroutine JACCOL in Appendix 1.

† The term appears in equation (4) and is computed as the array D(·,·) in subroutine YMAT. The listing may be found in Appendix 1.

3. Computational Complexity

The algorithm described in Section 2.1 makes use of a formula of Householder [11], which eliminates the need to invert the inaccessible nodal admittance matrix $\underline{Y}'$ to calculate the accessible transfer functions $\underline{y}$ each time a parameter is changed.† In this section we compare the amount of computation required to complete the search for $\min_{\underline{x}} ||\underline{M}(\underline{x})-\hat{\underline{M}}||$ using this efficient algorithm to the amount of computation which would be required if $\underline{Y}_n'$ were directly inverted at each step of the search.

The computational complexity will be described in terms of the number of complex multiplications performed by the algorithm. This depends on the network configuration and specifically on the number of accessible nodes, inaccessible nodes, branches, controlled sources, etc. The computational complexity of the efficient algorithm is even more difficult to estimate since it depends in addition, on such measures as the fraction of branches terminating on two inaccessible nodes, the fraction of branches terminating on only one inaccessible node, etc. For these reasons we have made the following assumptions: the total number of network parameters n is 1.1 times the number of branches, the number of branches is 2.2 times the number of inaccessible nodes, 45% of the branches terminate on 2 inaccessible nodes, 55% of the branches terminate on exactly 1 inaccessible node. Using these assumptions, the computational complexity can be calculated in terms of the number of inaccessible nodes, q.

The basic steps required to compute the measurable y-parameter matrix $\underline{y}$ are the same for both the direct and the efficient algorithms: (1) the new branch admittance matrix $\underline{Y}_b'$ is formed; (2) then the new nodal admittance matrix $\underline{Y}_n'$ is formed and inverted; (3) finally $\underline{y}$ is computed.

† See equations (4) - (7).

Steps (1) and (3) are identical in both the efficient and the direct methods. Making use of the assumptions discussed above, step (1) requires about $0.05q^2$ complex multiplications, and step (3) requires about $13.31q^2$ complex multiplications for a total of about $13.36q^2$ complex multiplications for steps (1) and (3) combined.

Using the efficient algorithm to invert $\underline{Y}_n'$ discussed in Section 2.1, step (2) requires about $q^2 + 3.45q + 4.21$ complex multiplications. Using the direct inversion of $\underline{Y}_n'$, step (2) requires about $5q^3 + 0.61q$ complex multiplications.

The total number of complex multiplications necessary to find the measurable y-parameters $\underline{y}$ after a single parameter is changed is therefore $14.36q^2 + 3.45q + 4.21$ using the efficient algorithm and $5q^3 + 13.36q^2 + 0.16q$ using the direct algorithm. The efficient algorithm is clearly very superior.

A Fibonacci search using 7 evaluations of $\underline{y}$ is performed for each parameter, and the number of parameters is assumed to be 1.1 times the number of branches or about 2.42q. Thus the number of complex multiplications required to search over all parameters is 16.94q times the values cited in the previous paragraph. These results are shown in Figure 4 as a function of the number of inaccessible nodes q.

We close this section with a discussion of the computational complexity of forming (20) and solving for the unknown parameters $\underline{x}$. Three steps are required:

(1) form the Jacobian matrix $\underline{F}$, which has a column for each parameter and a row for every measurement;

(2) form the product $\underline{F}.^T F$;

(3) solve for $\underline{x}$.

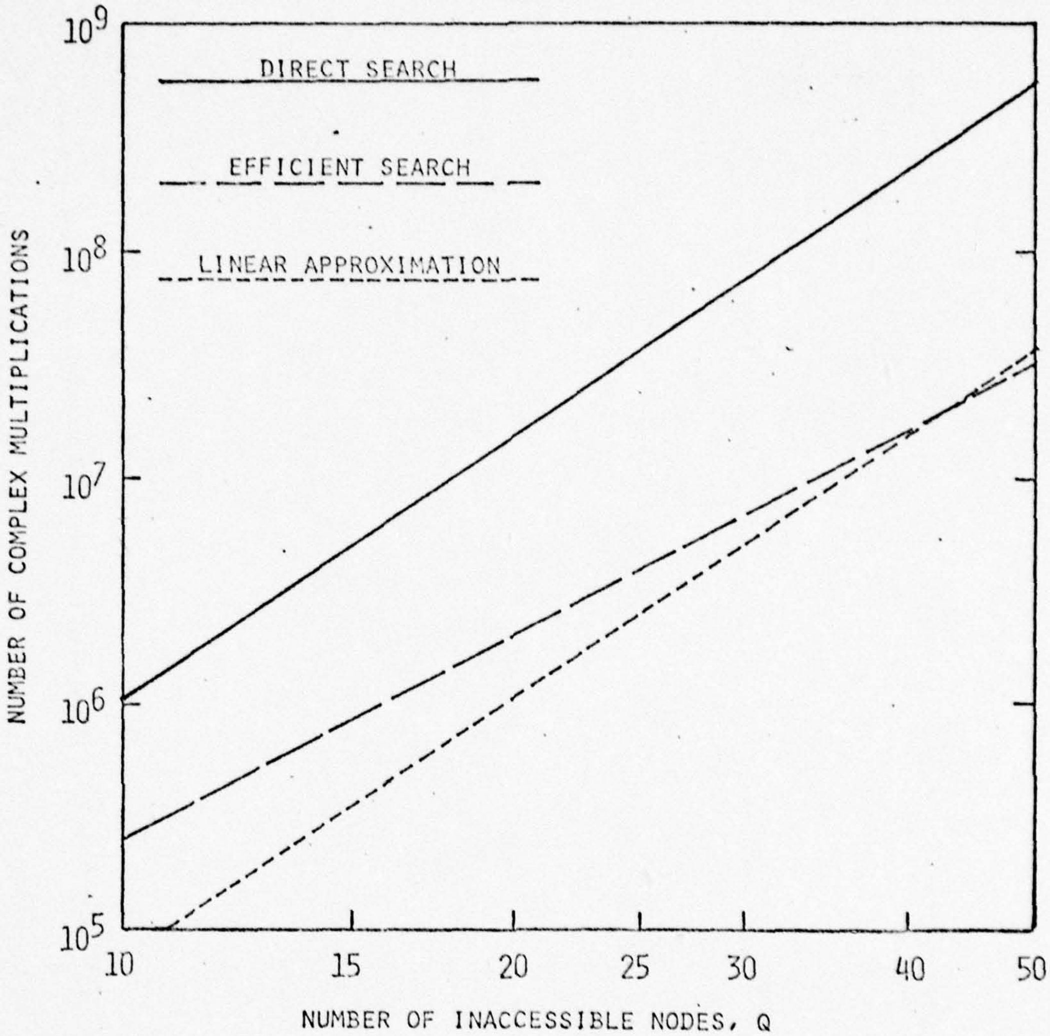

Figure 4 - The computational complexity of the parameter search.

Using the assumptions discussed previously, step (1) requires about $9.29q^3 + 26.14q^2$ real multiplications per parameter for a total of $22.48q^4 + 63.26q^3$ real multiplications. Step (2) requires about $2.93q^2m$ real multiplications, where m is the number of measurements. Assuming that $m = 2n \simeq 4.84q$, then step (2) requires about $14.18q^3$ real multiplications. Finally step (3) requires about $4.72q^3 + 5.86q^2 - 0.81q$ real multiplications. Therefore the total number of complex multiplications (1 complex multiplication = 4 real multiplications) to form (20) and solve for $\underline{x}$ is about $5.62q^4 + 20.54q^3 + 1.47q^2$. This is also plotted in Figure 4.

4.0 Conclusions

We have studied the problems of high resolution fault diagnosis in linear, analog electrical circuits in which a limited number of terminals are accessible for measurement. We have chosen to describe the faults in terms of short circuits between pairs of nodes and the values of parameters in the network model (resistances, capacitances, inductances, and controlled source gains). This approach is an outgrowth of our previous study [4], [5] in which the fault description was given only in terms of short circuits.

Specifically we have developed and tested an efficient algorithm for computing network responses (transfer functions at accessible terminals) after a single parameter is changed. The parameter change does not need to be small, and in fact the algorithm calculates exactly the new network responses, no matter how large the parameter change is. This algorithm was developed for use in a search over the network parameters for the network responses which best match the measured responses. The computational complexity (in terms of the number of complex multiplications) of this algorithm, which takes advantage of the special relationship between the network before and after a parameter is changed, was found to be very much better than the computational complexity of a direct algorithm. (See Figure 4).

In addition we studied the problem of choosing a set of measurements for fault analysis. We have found that the suitability of a set of measurements for resolving a fault can be expressed in terms of the Jacobian matrix $\underline{F}$ of the network responses (the partial derivatives of the network responses with respect to the network parameters). The fault can be resolved to the network parameter level only if the number of measurements m is at least as great as the number of network parameters n and, in addition, the rank of $\underline{F}$ is n. This latter condition can fail in the various ways illustrated in Section 2.2. The most promising way to find a set of independent measurements in practical cases appears to be to examine the numerical conditioning of the nxn matrix $\underline{F}^T\underline{F}$ over various combinations of measurements ($m \geq n$). We have written an efficient algorithm for forming the Jacobian matrix. Finding the "best-conditioned" nxn matrix $\underline{F}^T\underline{F}$ appears to be a time consuming process which seems best suited to a "simulation-before-test" process.

While investigating the measurement problem we discovered that it may be possible to accurately approximate the parameter set by solving a set of linear equations in which the coefficient matrix is $\underline{F}^T\underline{F}$. The computational complexity of forming and solving these equations increases as q^4 (q is the number of inaccessible nodes), whereas the computational complexity of an efficient search increases as q^3 . However, Figure 4 shows that one cycle of the linear approximation is less complex than one cycle of an efficient search (using a seven point Fibonacci search in each coordinate direction) for networks having fewer than 40 inaccessible nodes. Until testing is completed, however, it is impossible to judge how many cycles of the linear approximation are needed for a good solution, and problems of convergence are also possible.

5.0 Future Work

We have developed several efficient algorithms for computing network responses and partial derivatives, which can be put together in various ways to solve for the parameter values and short circuits, if any exist. Work is currently in progress to evaluate the various ways these algorithms can be used in forming an automatic fault diagnosis tool for analog electrical circuits which has the best combination of efficiency and reliability. This process will require a considerable amount of testing, including the use of real measurements on real circuits.

Appendix 1

Computer Program Listings

FAULT

```
ROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          MONDAY     10/08/79    01:43 PM
       /(E99000)  /FAULTO
       :$DYNAMIC 13000                                                      : 00000100             0FAULTO
       :$INTRINPACK 8                                                       : 00000200             0FAULTO
       :$SAVEFICM                                                           : 00000300             0FAULTO
       :$ERRORTRACE                                                         : 00000400             0FAULTO
       :FILE  3=FAULT3,UNIT=PRINTER                                         : 00000500             0FAULTO
       :FILE  5=FAULT3DATA,UNIT=DISK                                        : 00000600             0FAULTO
       :C                                                                   : 00000700             0FAULTO
       :C        THIS PROGRAM ISOLATES FAULTS IN ANALOG NETWORKS.  THE FAULT : 00000800            0FAULTO
       :C        IS ISOLATED BY COMPARING COMPUTED Y-PARAMETERS AT THE      : 00000900             0FAULTO
       :C        ACCESSIBLE NODES (BY A SEARCH OVER THE PARAMETER VALUES    : 00001000             0FAULTO
       :C        FOLLOWED BY SHORTING EVERY PAIR OF NODES INVOLVING AT LEAST : 00001100            0FAULTO
       :C        ONE INACCESSIBLE NODE) TO THE MEASURED Y-PARMETERS AT THE SAME : 00001200         0FAULTO
       :C        ACCESSIBLE NODES.  FOR A DESCRIPTION OF THE EFFICIENT      : 00001300             0FAULTO
       :C        ALGORITHM SEE:                                             : 00001400             0FAULTO
       :C                                                                   : 00001500             0FAULTO
       :C          A.T. JOHNSON, JR.,"EFFICIENT FAULT ANALYSIS IN ANALOG    : 00001600             0FAULTO
       :C             CIRCUITS", FINAL REPORT, USAF-ASEE SUMMER FACULTY RESEARCH : 00001700        0FAULTO
       :C             PROGRAM, WPAFB, OHIO, AUGUST 1978                     : 00001800             0FAULTO
       :C                                                                   : 00001900             0FAULTO
 0003  :         COMMON/DELP/  ITYPE,    Y2IP, COSTMN,    JPAR,    NPAR,  ICORR, : 00002000  000075  0FAULTO
 0003  :       *               IX, IW, W,     WROW,    WCOL,        X,    XROW,   XCOL : 00002100  000075  0FAULTO
 0004  :         COMPLEX   Y2IP(17,17), W(80), X(80)                        : 00002200  000075  0FAULTO
 0005  :         INTEGER   WROW(80), WCOL(80), XROW(80), XCOL(80)           : 00002300  000075  0FAULTO
 0006  :         INTEGER ITYPE(80), ICORR(80)                               : 00002400  000075  0FAULTO
       :C                                                                   : 00002500             0FAULTO
 0007  :      COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),      : 00002600  000075  0FAULTO
 0007  :      1             YB(65), YROW(65), GMROW(15), GMCOL(15),          : 00002700  000075  0FAULTO
 0007  :      2             RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),    : 00002800  000075  0FAULTO
 0007  :      3             AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,   : 00002900  000075  0FAULTO
 0007  :      4             IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR          : 00003000  000075  0FAULTO
 0008  :      INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,     : 00003100  000075  0FAULTO
 0008  :      1          AROW, ACOL, YBROW, YBCOL                            : 00003200  000075  0FAULTO
 0009  :      COMPLEX Y, YB                                                 : 00003300  000075  0FAULTO
       :C                                                                   : 00003400             0FAULTO
 0010  :      COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),  : 00003500  000075  0FAULTO
 0010  :      1            Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),  : 00003600  000075  0FAULTO
 0010  :      2            A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,    : 00003700  000075  0FAULTO
 0010  :      3            N, IB, ITOT, YN(3,3)                             : 00003800  000075  0FAULTO
 0011  :      INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                        : 00003900  000075  0FAULTO
 0012  :      INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                        : 00004000  000075  0FAULTO
 0013  :      COMPLEX YN, Y2, Y2I, Y2PI                                     : 00004100  000075  0FAULTO
       :C                                                                   : 00004200             0FAULTO
       :C                                                                   : 00004210             0FAULTO
 0014  :         COMMON/SRCH/ PAR, DELPAR, YNNOM                            : 00004220  000075  0FAULTO
       :C                                                                   : 00004230             0FAULTO
 0015  :      DIMENSION DENOM(3,3)                                          : 00004300  000075  0FAULTO
 0016  :      COMPLEX YNNOM(3,3), DELYN(3,3)                                : 00004400  000075  0FAULTO
       :C                                                                   : 00004600             0FAULTO
       :C        READ NETWORK DATA AND COMPUTE NODE-TO-BRANCH INCIDENCE MATRICES : 00004700         0FAULTO
       :C        (SPARSE STORAGE) OF ORIGINAL NETWORK, A1PLUS, A1MNUS, A2PLUS, : 00004800          0FAULTO
       :C        A2MNUS.                                                    : 00004900             0FAULTO
       :C                                                                   : 00005000             0FAULTO
 0017  :      CALL INPUT                                                    : 00005100  000075  0FAULTO
       :C                                                                   : 00007000             0FAULTO
       :C        READ MEASURED Y-PARAMETERS: YNNOM                          : 00007100             0FAULTO
       :C                                                                   : 00007200             0FAULTO
 0018  :      DO 5 K=1,IP                                                   : 00007300  000592  0FAULTO
 0019  :      DO 5 L=1,IP                                                   : 00007400  000683  0FAULTO
 0020  :      READ(5,108) YNNOM(K,L)                                        : 00007500  000774  0FAULTO
 0021  :      DENOM(K,L) = CABS(YNNOM(K,L))                                 : 00007600  001286  0FAULTO
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0022   :       IF(DENOM(K,L) .EQ. 0.) DENOM(K,L) = 1.E-9                    : 00007700   001846   OFAULTO
0024   :     5 CONTINUE                                                     : 00007800   002361   OFAULTO
0025   :       WRITE(3,109)                                                 : 00007900   002706   OFAULTO
0026   :       DO 1 M=1,IP                                                  : 00008000   002981   OFAULTO
0027   : 1     WRITE(3,100)(YNNOM(M,NX),NX=1,IP)                            : 00008100   003072   OFAULTO
0028   :       IQ= N-IP                                                     : 00008200   003996   OFAULTO
       :C                                                                   : 00008225            OFAULTO
       :C                                                                   : 00008300            OFAULTO
       :C        COMPUTE YB MATRIX. SEE REF., EQ.(3).                       : 00008400            OFAULTO
       :C                                                                   : 00008500            OFAULTO
0029   :       CALL YBMAT                                                   : 00008600   004144   OFAULTO
       :C                                                                   : 00008700            OFAULTO
       :C        COMPUTE Y2 = A2 * YB * A2T.  SEE REF., EQ.(2).             : 00008800            OFAULTO
       :C                                                                   : 00008900            OFAULTO
0030   :       CALL Y2PMAT(Y2,IQ,17,A2PLUS,A2MNUS)                          : 00009000   004249   OFAULTO
       :C                                                                   : 00009100            OFAULTO
       :C        COMPUTE INVERSE OF Y2.                                     : 00009200            OFAULTO
       :C                                                                   : 00009300            OFAULTO
0031   :         IF(IQ .NE. 1) GO TO 8                                      : 00009320   004664   OFAULTO
0033   :           Y2I(1,1) = (1.,0.)/Y2(1,1)                               : 00009340   004797   OFAULTO
0034   :           GO TO 7                                                  : 00009360   005252   OFAULTO
0035   :     8 CALL MINVC(Y2,Y2I,IQ,17)                                     : 00009400   005338   OFAULTO
       :C                                                                   : 00009500            OFAULTO
       :C        COMPUTE Y-PARAMETERS AT ACCESSIBLE TERMINALS. REF., EQ.(8). : 00009600           OFAULTO
       :C                                                                   : 00009700            OFAULTO
0036   :     7 CALL YMAT(A1PLUS,A1MNUS,A2PLUS,A2MNUS,Y2I,YN,IP,IB,17)       : 00009800   005753   OFAULTO
0037   :       ISAVE = 0                                                    : 00009900   006456   OFAULTO
0038   :       JSAVE = 0                                                    : 00010000   006547   OFAULTO
       :C                                                                   : 00010100            OFAULTO
       :C        PRINT Y-PARAMETERS OF UNFAULTED NETWORK WITH NOMINAL       : 00010200            OFAULTO
       :C        PARAMETER VALUES.                                          : 00010300            OFAULTO
       :C                                                                   : 00010400            OFAULTO
0039   :       WRITE(3,102)                                                 : 00010500   006638   OFAULTO
0040   :       DO 2 M=1,IP                                                  : 00010600   006911   OFAULTO
0041   : 2     WRITE(3,100)(YN(M,NX),NX=1,IP)                               : 00010700   007002   OFAULTO
       :C                                                                   : 00010800            OFAULTO
       :C        COMPUTE COST FUNCTION OF UNFAULTED NETWORK.  CALL IT COSTMN. : 00010900          OFAULTO
       :C                                                                   : 00011000            OFAULTO
0042   :       COSTMN = COSTF(YNNOM)                                        : 00011100   007926   OFAULTO
0043   :       WRITE(3,107) COSTMN                                          : 00011200   008187   OFAULTO
       :C                                                                   : 00011400            OFAULTO
       :C        COMPUTE AND PRINT PARTIAL DERIVITIVES OF Y-PARAMETERS WITH : 00011500            OFAULTO
       :C        RESPECT TO EACH NETWORK PARAMETER.                         : 00011600            OFAULTO
       :C                                                                   : 00011700            OFAULTO
0044   :         SENMAX = 0.                                                : 00011750   008485   OFAULTO
0045   :         DO 3 I = 1,IPAR                                            : 00011800   008613   OFAULTO
0046   :           WRITE(3,110) I                                           : 00011850   008704   OFAULTO
0047   :           SENS = 0.                                                : 00011900   009002   OFAULTO
0048   :           CALL JACCOL(DELYN, I)                                    : 00012000   009130   OFAULTO
0049   :           DO 4 K = 1,IP                                            : 00012010   009397   OFAULTO
0050   :           DO 6 L = 1,IP                                            : 00012020   009488   OFAULTO
0051   :             RY1 = REAL( YN(K,L) )                                  : 00012030   009579   OFAULTO
0052   :             XY1 = AIMAG( YN(K,L) )                                 : 00012040   009934   OFAULTO
0053   :             RY2 = REAL( YNNOM(K,L) )                               : 00012050   010298   OFAULTO
0054   :             XY2 = AIMAG( YNNOM(K,L) )                              : 00012060   010653   OFAULTO
0055   :                   IF(RY2 .EQ. 0.) RY2 = 1.                         : 00012062   011017   OFAULTO
0057   :                   IF(XY2 .EQ. 0.) XY2 = 1.                         : 00012064   011304   OFAULTO
0058   :             SENS = SENS + (RY1 - RY2)*REAL( DELYN(K,L) )/(RY2*RY2) : 00012070   011591   OFAULTO
0059   :    *              + (XY1 - XY2)*AIMAG( DELYN(K,L) )/(XY2*XY2)       : 00012080   011591   OFAULTO
0060   :     6       CONTINUE                                               : 00012082   012536   OFAULTO
0061   :         WRITE(3,100)(DELYN(K,NX), NX=1,IP)                         : 00012084   012739   OFAULTO
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    4     CONTINUE
          IF(ITYPE(I) .EQ. 1) GO TO 520
          IF(ITYPE(I) .GE. 4) GO TO 525
            IF(AIMAG( Y(ICORR(I)) ) .LT. 0.) GO TO 510
              ELE = AIMAG(Y(ICORR(I)))/OMEGA
            GO TO 530
  510       ELE = -1./(OMEGA*AIMAG(Y(ICORR(I))))
            GO TO 530
  520       ELE = REAL(Y(ICORR(I)))
            GO TO 530
  525     GO TO (526,527,528,529), ITYPE(I) - 10
  526     ELE = GM(ICORR(I))
          GO TO 530
  527     ELE = RM(ICORR(I))
          GO TO 530
  528     ELE = ALPHA(ICORR(I))
          GO TO 530
  529     ELE = XMU(ICORR(I))
  530     SENS = 2.*SENS*ELE
          WRITE(3,104) I, SENS
          IF(ABS(SENMAX) .GT. ABS(SENS)) GO TO 3
            SENMAX = SENS
            IPSAVE = I
    3   CONTINUE
        WRITE(3,105) SENMAX, IPSAVE
  104   FORMAT(///10X, '(ERROR SENSITIVITY WITH RESPECT TO PARAMETER',
     *         I3, ') =', E11.3)
  105   FORMAT(///10X, 'MAXIMUM SENSITIVITY =', E11.3,
     *         ' WITH RESPECT TO PARAMETER', I3)
C
C       *** BEGIN PARAMETER SEARCH ***
C
        DO 41 I = 1, IPAR
        J = ICORR(I)
        NPAR = J
        IF(ITYPE(I) .GE. 4) GO TO 35
        JPAR = 1
        IF(ITYPE(I) .NE. 1) GO TO 31
          PAR = REAL( Y(J) )
        GO TO 40
   31     PAR = AIMAG( Y(J) )
        GO TO 40
   35   JPAR = ITYPE(I) - 10
        IF(JPAR .EQ. 1) JPAR = 5
        GO TO(36, 37, 38, 39), JPAR - 1
   36   PAR = RM(J)
        GO TO 40
   37   PAR = ALPHA(J)
        GO TO 40
   38   PAR = XMU(J)
        GO TO 40
   39   PAR = GM(J)
   40   CALL SEARCH( 0., 1.1*PAR, 0.05*PAR, .TRUE., PAR, COSTMX)
         COSTMN = 1./COSTMX
         DO 41 NX = 1,10
         DO 41 NY = 1,10
   41   Y2I(NX,NY) = Y2IP(NX,NY)
C
C       PRINT LOWEST COST AND NEW PARAMETER VAUES AFTER PARAMETER
C       SEARCH.
C
        WRITE(3,111) COSTMN
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0122 :     WRITE(3,106)                                          : 00016020   024436   OFAULTO
0123 :     DO 43 I = 1,IP                                        : 00016040   024709   OFAULTO
0124 :  43 WRITE(3,100) (YN(I,NX), NX = 1,IP)                    : 00016060   024800   OFAULTO
0125 :     WRITE(3,112)                                          : 00016100   025724   OFAULTO
0126 :     DO 50 I = 1,IB                                        : 00016200   025997   OFAULTO
0127 :        IF(ITYPE(I) .EQ. 1) GO TO 60                       : 00016300   026083   OFAULTO
0129 :        IF(AIMAG(Y(I)) .LT. 0.) GO TO 55                   : 00016400   026284   OFAULTO
     :C                                                          : 00016500            OFAULTO
     :C       BRANCH I IS A CAPACITOR: ABS(Y) = OMEGA*C          : 00016600            OFAULTO
     :C                                                          : 00016700            OFAULTO
0131 :        ELE = AIMAG(Y(I))/OMEGA                            : 00016800   026641   OFAULTO
0132 :        LABEL = 'C = '                                     : 00016900   026987   OFAULTO
0133 :        DIM = ' F  '                                       : 00017000   027115   OFAULTO
0134 :        GO TO 70                                           : 00017100   027243   OFAULTO
     :C                                                          : 00017200            OFAULTO
     :C       BRANCH I IS AN INDUCTOR: ABS(Y) = 1/(OMEGA*L)      : 00017300            OFAULTO
     :C                                                          : 00017400            OFAULTO
0135 :  55    ELE = -1./(OMEGA*AIMAG(Y(I)))                      : 00017500   027329   OFAULTO
0136 :        LABEL = 'L = '                                     : 00017600   027723   OFAULTO
0137 :        DIM = ' H  '                                       : 00017700   027851   OFAULTO
0138 :        GO TO 70                                           : 00017800   027979   OFAULTO
     :C                                                          : 00017900            OFAULTO
     :C       BRANCH I IS A RESISTOR: Y = 1/R                    : 00018000            OFAULTO
     :C                                                          : 00018100            OFAULTO
0139 :  60    ELE = 1./REAL(Y(I))                                : 00018200   028065   OFAULTO
0140 :        LABEL = 'R = '                                     : 00018300   028416   OFAULTO
0141 :        DIM = ' OHM'                                       : 00018400   028544   OFAULTO
0142 :  70 WRITE(3,113) ICORR(I), LABEL, ELE, DIM                : 00018500   028672   OFAULTO
0143 :     IF(IRM .EQ. 0) GO TO 163                              : 00018600   029435   OFAULTO
0145 :     DO 62 K = 1,IRM                                       : 00018700   029568   OFAULTO
0146 :        IF(RMROW(K) .EQ. I) GO TO 82                       : 00018800   029659   OFAULTO
0148 :  62 CONTINUE                                              : 00018900   029867   OFAULTO
0149 : 163 IF(IALPHA .EQ. 0) GO TO 164                           : 00019000   030070   OFAULTO
0151 :     DO 63 K = 1,IALPHA                                    : 00019100   030203   OFAULTO
0152 :        IF(AROW(K) .EQ. I) GO TO 83                        : 00019200   030294   OFAULTO
0154 :  63 CONTINUE                                              : 00019300   030502   OFAULTO
0155 : 164 IF(IXMU .EQ. 0) GO TO 165                             : 00019400   030705   OFAULTO
0157 :     DO 64 K = 1,IXMU                                      : 00019500   030838   OFAULTO
0158 :        IF(XMUROW(K) .EQ. I) GO TO 84                      : 00019600   030929   OFAULTO
0160 :  64 CONTINUE                                              : 00019700   031137   OFAULTO
0161 : 165 IF(IGM .EQ. 0) GO TO 50                               : 00019800   031340   OFAULTO
0163 :     DO 65 K = 1,IGM                                       : 00019900   031473   OFAULTO
0164 :        IF(GMROW(K) .EQ. I) GO TO 85                       : 00020000   031564   OFAULTO
0166 :  65 CONTINUE                                              : 00020100   031772   OFAULTO
0167 :     GO TO 50                                              : 00020200   031975   OFAULTO
0168 :  82 SOURCE = 'RM ='                                       : 00020300   032061   OFAULTO
0169 :     DIM = ' OHM'                                          : 00020400   032189   OFAULTO
0170 :     GAIN = RM(K)                                          : 00020500   032317   OFAULTO
0171 :     GO TO 51                                              : 00020600   032494   OFAULTO
0172 :  83 SOURCE = 'AL ='                                       : 00020700   032580   OFAULTO
0173 :     GAIN = ALPHA(K)                                       : 00020800   032708   OFAULTO
0174 :     DIM = '    '                                          : 00020900   032885   OFAULTO
0175 :     GO TO 51                                              : 00021000   033013   OFAULTO
0176 :  84 SOURCE = 'MU ='                                       : 00021100   033099   OFAULTO
0177 :     GAIN = XMU(K)                                         : 00021200   033227   OFAULTO
0178 :     DIM = '    '                                          : 00021300   033404   OFAULTO
0179 :     GO TO 51                                              : 00021400   033532   OFAULTO
0180 :  85 SOURCE = 'GM ='                                       : 00021500   033618   OFAULTO
0181 :     GAIN = GM(K)                                          : 00021600   033746   OFAULTO
0182 :     DIM = ' MHO'                                          : 00021700   033923   OFAULTO
0183 :  51 WRITE(3,114) SOURCE, GAIN, DIM                        : 00021800   034051   OFAULTO
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0184  :     50 CONTINUE                                                         : 00021900   034617  OFAULTD
      :C                                                                        : 00022000           OFAULTD
      :C          LOOP THROUGH ALL PAIRS OF SHORT CIRCUITS INVOLVING AN         : 00022100           OFAULTD
      :C          INACCESSIBLE NODE.  I AND J ARE THE SHORTED NODES.            : 00022200           OFAULTD
      :C                                                                        : 00022300           OFAULTD
0185  :          DO 10 K = IP+1,N                                               : 00022400   034820  OFAULTD
0186  :          J = N + IP + 1 - K                                             : 00022500   034956  OFAULTD
0187  :          NCASES = NCASES - J                                            : 00022600   035160  OFAULTD
0188  :          DO 10 L=1,J                                                    : 00022700   035308  OFAULTD
0189  :          I = L-1                                                        : 00022800   035399  OFAULTD
      :C                                                                        : 00022900           OFAULTD
      :C          COMPUTE NEW NODE-TO-BRANCH INCIDENCE MATRICES (A1PLSP, A1MNSP, : 00023000          OFAULTD
      :C          A2PLSP, A2MNSP) WITH NODES I AND J SHORTED.  SEE REF., EQ. (13).: 00023100         OFAULTD
      :C                                                                        : 00023200           OFAULTD
0190  :          CALL ANEW                                                      : 00023300   035535  OFAULTD
      :C                                                                        : 00023400           OFAULTD
      :C          COMPUTE INVERSE OF Y2P FOR FAULTED NETWORK.  SEE REF., EQ.(10). : 00023500         OFAULTD
      :C                                                                        : 00023600           OFAULTD
0191  :          IF(IQ .EQ. 1) GO TO 52                                         : 00023650   035640  OFAULTD
0193  :          CALL SNEW                                                      : 00023700   035773  OFAULTD
      :C                                                                        : 00023800           OFAULTD
      :C          COMPUTE Y-PARAMETERS AT ACCESSIBLE TERMINALS OF FAULTED       : 00023900           OFAULTD
      :C          NETWORK.  SEE REF., EQ.(23).                                  : 00024000           OFAULTD
      :C                                                                        : 00024100           OFAULTD
0194  :     52 CALL YMAT(A1PLSP,A1MNSP,A2PLSP,A2MNSP,Y2PI,YN,IP,IB,16)          : 00024200   035878  OFAULTD
      :C                                                                        : 00024300           OFAULTD
      :C          COMPUTE COST FUNCTION OF CURRENT Y-PARAMETERS (NODES I AND J  : 00024400           OFAULTD
      :C          SHORTED).                                                     : 00024500           OFAULTD
      :C                                                                        : 00024600           OFAULTD
0195  :          COST = COSTF(YNNOM)                                            : 00024700   036581  OFAULTD
0196  :          IF(COST - COSTMN) 22,21,21                                     : 00024800   036842  OFAULTD
0197  :     22   COSTMN = COST                                                  : 00024900   037023  OFAULTD
0198  :          ISAVE = I                                                      : 00025000   037131  OFAULTD
0199  :          JSAVE = J                                                      : 00025100   037239  OFAULTD
      :C                                                                        : 00025200           OFAULTD
      :C          PRINT NODES I AND J AND THE COMPUTED Y-PARAMETERS AT THE      : 00025300           OFAULTD
      :C          ACCESSIBLE TERMINALS.                                         : 00025400           OFAULTD
      :C                                                                        : 00025500           OFAULTD
0200  :     21   WRITE(3,101) I,J                                               : 00025600   037347  OFAULTD
0201  :          DO 30 M=1,IP                                                   : 00025700   037779  OFAULTD
0202  :     30   WRITE(3,100) (YN(M,NX),NX=1,IP)                                : 00025800   037870  OFAULTD
0203  :     10   WRITE(3,107) COST                                              : 00025900   038794  OFAULTD
      :C                                                                        : 00026000           OFAULTD
      :C          PRINT NODES OF LOWEST COST FAULT AND COST FUNCTION FOR        : 00026100           OFAULTD
      :C          THIS PAIR OF SHORTED NODES.                                   : 00026200           OFAULTD
      :C                                                                        : 00026300           OFAULTD
0204  :          WRITE(3,103) ISAVE, JSAVE, COSTMN                              : 00026400   039380  OFAULTD
0205  :          STOP                                                           : 00026500   039946  OFAULTD
0206  :    100   FORMAT (3(10X,2E12.4,' J')//)                                  : 00026600   040064  OFAULTD
0207  :    101   FORMAT('1',10X,'Y-PARAMETERS WITH NODES',I3,                   : 00026700   040064  OFAULTD
0207  :        *   ' AND',I3,' SHORTED'///)                                     : 00026800   040064  OFAULTD
0208  :    102   FORMAT(///,10X,'BEGIN SEARCH USING NOMINAL NETWORK.'///,       : 00026900   040064  OFAULTD
0208  :        *      10X, 'Y-PARAMETERS WITH NO SHORT CIRCUITS, MHOS:'/)       : 00027000   040064  OFAULTD
0209  :    103   FORMAT('1',10X,'MOST LIKELY SINGLE SHORT:'//11X,'NODES',       : 00027100   040064  OFAULTD
0209  :        *          I3,' AND',I4,' SHORTED.'///10X,'(COST FUNCTION =',    : 00027200   040064  OFAULTD
0209  :        *          E11.4,')' )                                           : 00027300   040064  OFAULTD
0210  :    106   FORMAT(/30X, 'Y-PARAMETERS'/)                                  : 00027325   040064  OFAULTD
0211  :    107   FORMAT(//,56X,'COST =', E11.3)                                 : 00027400   040064  OFAULTD
0212  :    108   FORMAT(E10.4,1X,E10.4)                                         : 00027500   040064  OFAULTD
0213  :    109   FORMAT(///,10X,'MEASURED Y-PARAMETERS, MHOS.'/)                : 00027600   040064  OFAULTD
0214  :    110   FORMAT(5(/), 10X, 'PARTIAL DERIVATIVES OF Y-PARAMETERS',       : 00027625   040064  OFAULTD
0214  :        *         ' WITH RESPECT TO NETWORK PARAMETER', I3/)             : 00027650   040064  OFAULTD
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0215  : 111  FORMAT(//, 41X, 'LOWEST COST AFTER PARAMETER SEARCH =',        : 00027700   040064  OFAULTO
0215  :      *         E11.3)                                                : 00027800   040064  OFAULTO
0216  : 112  FORMAT(//, 9X, 'BRANCH', T22, 'PASSIVE', T43, 'ACTIVE',         : 00027900   040064  OFAULTO
0216  :      *         /, 9X, 'NUMBER', T22, 'ELEMENT', T43, 'ELEMENT'/)     : 00028000   040064  OFAULTO
0217  : 113  FORMAT( I14, 3X, A4, E10.4, A4 )                                : 00028100   040064  OFAULTO
0218  : 114  FORMAT( '+', T39, A4, E10.4, A4 )                               : 00028200   040064  OFAULTO
0219  : 200  FORMAT(3(F8.2,F6.2,' J'))                                       : 00028300   040064  OFAULTO
0220  :      END                                                             : 00028400   040064  OFAULTO
0 ERRORS AND NO WARNINGS IN 220 STATEMENTS,CODE EMITTED = 40182 BITS (5023 BYTES)OFAULTO
COMPILE TIME IS 7 MINUTES AND 44.9 SECONDS FOR 314 CARDS AT 41 CARDS/MINUTE.
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BURROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79    05:18 PM
         /(E99000)   /FAULT30
    0001   :          BLOCK DATA                                                           : 00028500    000075   OFAULT30
    0002   :          COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),         : 00028600    000075   BLOCK.
    0002   :         1              Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),        : 00028700    000075   BLOCK.
    0002   :         2              A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,          : 00028800    000075   BLOCK.
    0002   :         3              N, IB, ITOT, YN(3,3)                                   : 00028900    000075   BLOCK.
    0003   :          INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                               : 00029000    000075   BLOCK.
    0004   :          INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                               : 00029100    000075   BLOCK.
    0005   :          DATA A2PLUS,A2MNUS/100*0/                                            : 00029200    000075   BLOCK.
    0006   :          DATA A1PLUS,A1MNUS/100*0/                                            : 00029300    000075   BLOCK.
    0007   :          END                                                                  : 00029400    000075   BLOCK.

NO ERRORS AND NO WARNINGS IN 7 STATEMENTS,CODE EMITTED = 258 BITS (33 BYTES)BLOCK.
COMPILE TIME IS 26.9 SECONDS FOR 10 CARDS AT 22 CARDS/MINUTE.
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URROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79    05:18 PM
    /(E99000)  /FAULT30
 0001   :     SUBROUTINE INPUT                                                 : 00029500  000075  OFAULT30
        :C                                                                     : 00029600          OFAULT30
        :C      THIS SUBROUTINE READS THE NETWORK DATA AND STORES THE NODE-   : 00029700          OFAULT30
        :C      TO-BRANCH INCIDENCE MATRICES.                                  : 00029800          OFAULT30
        :C                                                                     : 00029900          OFAULT30
        :C      VARIABLE DEFINITIONS:                                          : 00030000          OFAULT30
        :C                                                                     : 00030100          OFAULT30
        :C        IB      - NUMBER OF BRANCHES IN NETWORK (PASSIVE ELEMENTS)  : 00030200          OFAULT30
        :C        NB      - BRANCH NUMBER                                      : 00030300          OFAULT30
        :C        IPAR    - PARAMETER NUMBER, ASSIGNED SEQUENTIALLY IN THE     : 00030400          OFAULT30
        :C                     ORDER PARAMETERS ARE READ.                      : 00030500          OFAULT30
        :C        ITYPE(.)      VECTOR OF PARAMETER TYPES: 1 = RESISTOR,       : 00030600          OFAULT30
        :C                      2 = INDUCTOR, 3 = CAPACITOR, 11 = GM,          : 00030700          OFAULT30
        :C                      12 = RM, 13 = ALPHA, 14 = MU                   : 00030800          OFAULT30
        :C        ICORR(IPAR) = BRANCH NUMBER OF PARAMETER IPAR                : 00030900          OFAULT30
        :C                                                                     : 00031000          OFAULT30
        :C                                                                     : 00031100          OFAULT30
        :C        IGM     - NUMBER OF TRANSCONDUCTANCES                        : 00031200          OFAULT30
        :C        IRM     - NUMBER OF TRANSRESISTANCES                         : 00031300          OFAULT30
        :C        IYMU    - NUMBER OF VOLTAGE GAINS                            : 00031400          OFAULT30
        :C        IALPHA  - NUMBER OF CURRENT GAINS                            : 00031500          OFAULT30
        :C        N       - NUMBER OF NODES (NOT COUNTING REFERENCE NODE)      : 00031600          OFAULT30
        :C        IP      - NUMBER OF ACCESSIBLE NODES ( " )                   : 00031700          OFAULT30
        :C        FREQ    - FREQUENCY AT WHICH ANALYSIS IS PERFORMED (HZ)      : 00031800          OFAULT30
        :C        Y       - MATRIX OF PASSIVE ELEMENT ADMITTANCES (SPARSE)     : 00031900          OFAULT30
        :C        GM      - MATRIX OF TRANSCONDUCTANCES (SPARSE)               : 00032000          OFAULT30
        :C        RM      - MATRIX OF TRANSRESISTANCES (SPARSE)                : 00032100          OFAULT30
        :C        XMU     - MATRIX OF VOLTAGE GAINS (SPARSE)                   : 00032200          OFAULT30
        :C        ALPHA   - MATRIX OF CURRENT GAINS (SPARSE)                   : 00032300          OFAULT30
        :C        YROW(.)  VECTOR STORING ROW OF Y(.)                          : 00032400          OFAULT30
        :C        GMROW(.) VECTOR STORING ROW OF GM(.)                         : 00032500          OFAULT30
        :C        GMCOL(.) VECTOR STORING COLUMN OF GM(.)                      : 00032600          OFAULT30
        :C                                                                     : 00032700          OFAULT30
        :C           SIMILARLY FOR RMROW, RMCOL, XMUROW, XMUCOL, AROW, ACOL.   : 00032800          OFAULT30
        :C                                                                     : 00032900          OFAULT30
 0002   :       COMMON/CELP/  ITYPE,    Y2IP, COSTMN,    JPAR,    NPAR, ICORR, : 00033000  000075  INPUT
 0002   :    *          IX, IW,    W,    WROW,    WCOL,       X,   XROW,   XCOL: 00033100  000075  INPUT
 0003   :       COMPLEX   Y2IP(17,17), W(80), X(80)                            : 00033200  000075  INPUT
 0004   :       INTEGER   WROW(80), WCOL(80), XROW(80), XCOL(80)               : 00033300  000075  INPUT
 0005   :       INTEGER   ITYPE(80), ICORR(80)                                 : 00033400  000075  INPUT
        :C                                                                     : 00033500          INPUT
 0006   :     COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),         : 00033600  000075  INPUT
 0006   :    1               YB(65), YROW(65), GMROW(15), GMCOL(15),           : 00033700  000075  INPUT
 0006   :    2               RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),     : 00033800  000075  INPUT
 0006   :    3               AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,    : 00033900  000075  INPUT
 0006   :    4               IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR           : 00034000  000075  INPUT
 0007   :     INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,        : 00034100  000075  INPUT
 0007   :    1         AROW, ACOL, YBROW, YBCOL                                : 00034200  000075  INPUT
 0008   :     COMPLEX Y, YB                                                    : 00034300  000075  INPUT
 0009   :     COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),     : 00034400  000075  INPUT
 0009   :    1              Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),    : 00034500  000075  INPUT
 0009   :    2              A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,      : 00034600  000075  INPUT
 0009   :    3              N, IB, ITOT, YN(3,3)                               : 00034700  000075  INPUT
 0010   :     INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                           : 00034800  000075  INPUT
 0011   :     INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                           : 00034900  000075  INPUT
 0012   :     COMPLEX YN, Y2, Y2I, Y2PI                                        : 00035000  000075  INPUT
 0013   :     READ(5,100) IP,N,FREQ                                            : 00035100  000075  INPUT
 0014   :     OMEGA = 2.*3.141593*FREQ                                         : 00035200  000799  INPUT
 0015   :     IB = 0                                                           : 00035300  001009  INPUT
```

```
      IPAR = 0                                              00035400   001100  INPUT
      IGM = 0                                               00035500   001191  INPUT
      IRM = 0                                               00035600   001282  INPUT
      IXMU = 0                                              00035700   001373  INPUT
      IALPHA = 0                                            00035800   001464  INPUT
      WRITE(3,102) N, IP, FREQ                              00035900   001555  INPUT
    1 READ(5,101)NB,N1,N2,ITYP,VALUE                        00036000   002121  INPUT
      IF(NB .EQ. 99999) GO TO 200                           00036100   002965  INPUT
      WRITE(3,106)NB,N1,N2,ITYP,VALUE                       00036200   003124  INPUT
      IF (NB - 10000) 10,20,20                              00036300   003958  INPUT
   10 IB = IB+1                                             00036400   004153  INPUT
         IPAR = IPAR + 1                                    00036500   004289  INPUT
         ITYPE(IPAR) = ITYP                                 00036600   004425  INPUT
         ICORR(IPAR) = IB                                   00036700   004600  INPUT
      IF(N1 .EQ. 0) GO TO 15                                00036800   004775  INPUT
      IF (N1-IP) 16,16,17                                   00036900   004908  INPUT
   16 A1PLUS(NB) = N1                                       00037000   005089  INPUT
      GO TO 15                                              00037100   005264  INPUT
   17 A2PLUS(NB) = N1-IP                                    00037200   005350  INPUT
   15 IF(N2.EQ.0) GO TO 18                                  00037300   005559  INPUT
      IF(N2-IP) 19,19,30                                    00037400   005692  INPUT
   19 A1MNUS(NB) = N2                                       00037500   005873  INPUT
      GO TO 18                                              00037600   006048  INPUT
   30 A2MNUS(NB) = N2-IP                                    00037700   006134  INPUT
   18 GO TO (11,12,13), ITYP                                00037800   006343  INPUT
   11 Y(NB) = CMPLX(1./VALUE, 0.)                           00037900   006498  INPUT
      GO TO 2                                               00038000   006905  INPUT
   12 Y(NB) = (0.,-1.)/(OMEGA*VALUE)                        00038100   006991  INPUT
      GO TO 2                                               00038200   007589  INPUT
   13 Y(NB) = (0.,1.)*OMEGA*VALUE                           00038300   007675  INPUT
    2 YROW(NB) = NB                                         00038400   008218  INPUT
      GO TO 1                                               00038500   008393  INPUT
   20 IPAR = IPAR + 1                                       00038600   008479  INPUT
      ITYPE(IPAR) = ITYP + 10                               00038700   008615  INPUT
      GO TO (21,22,23,24), ITYP                             00038800   008812  INPUT
   21 IGM = IGM + 1                                         00038900   008985  INPUT
      ICORR(IPAR) = IGM                                     00039000   009121  INPUT
      GM(IGM) = VALUE                                       00039100   009296  INPUT
      GMROW(IGM) = N1                                       00039200   009471  INPUT
      GMCOL(IGM) = N2                                       00039300   009646  INPUT
      GO TO 1                                               00039400   009821  INPUT
   22 IRM = IRM + 1                                         00039500   009907  INPUT
      ICORR(IPAR) = IRM                                     00039600   010043  INPUT
      RM(IRM) = VALUE                                       00039700   010218  INPUT
      RMROW(IRM) = N1                                       00039800   010393  INPUT
      RMCOL(IRM) = N2                                       00039900   010568  INPUT
      GO TO 1                                               00040000   010743  INPUT
   23 IALPHA = IALPHA + 1                                   00040100   010829  INPUT
      ICORR(IPAR) = IALPHA                                  00040200   010965  INPUT
      ALPHA(IALPHA) = VALUE                                 00040300   011140  INPUT
      AROW(IALPHA) = N1                                     00040400   011315  INPUT
      ACOL(IALPHA) = N2                                     00040500   011490  INPUT
      GO TO 1                                               00040600   011665  INPUT
   24 IXMU = IXMU + 1                                       00040700   011751  INPUT
      ICORR(IPAR) = IXMU                                    00040800   011887  INPUT
      XMU(IXMU) = VALUE                                     00040900   012062  INPUT
      XMUROW(IXMU) = N1                                     00041000   012237  INPUT
      XMUCOL(IXMU) = N2                                     00041100   012412  INPUT
      GO TO 1                                               00041200   012587  INPUT
  200 ITOT = IGM + IRM + IALPHA + IXMU                      00041300   012673  INPUT
      WRITE(3,105) IB, ITOT                                 00041400   012889  INPUT
      WRITE(3,107) IGM, IRM, IALPHA, IXMU                   00041500   013321  INPUT
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0081  :       RETURN                                                                      : 00041600   014021  INPUT
0082  : 100   FORMAT (2I5,E10.5)                                                          : 00041700   014135  INPUT
0083  : 101   FORMAT (4I5,E10.5)                                                          : 00041800   014135  INPUT
0084  : 102   FORMAT('1',25X,'NETWORK DATA:'//16X,I3,' NODES'/16X,I3,' ACCESSIBL: 00041900   014135  INPUT
0084  :      *E NODES'//16X,'ANALYSIS AT',E12.4,' HZ',10(/),2X,'NETWORK DATA CAR: 00042000   014135  INPUT
0084  :      *DS'//)                                                                      : 00042100   014135  INPUT
0085  : 105   FORMAT(//25X,'NETWORK INPUT COMPLETE:',I3,' BRANCHES,',                     : 00042200   014135  INPUT
0085  :      1        I4,' CONTROLLED SOURCES INCLUDING:'//)                              : 00042300   014135  INPUT
0086  : 106   FORMAT(20X,I3,3I5,E16.7)                                                    : 00042400   014135  INPUT
0087  : 107   FORMAT(I60,' TRANSCONDUCTANCES'/I60,' TRANSRESISTANCES'/                    : 00042500   014135  INPUT
0087  :      1        I60,' CURRENT GAINS'/I60,' VOLTAGE GAINS')                          : 00042600   014135  INPUT
0088  :       END                                                                         : 00042700   014135  INPUT

NO ERRORS AND NO WARNINGS IN 88 STATEMENTS,CODE EMITTED = 14249 BITS (1782 BYTES)INPUT
COMPILE TIME IS 50.5 SECONDS FOR 133 CARDS AT 158 CARDS/MINUTE.
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0001   :         SUBROUTINE YBMAT                                          : 00042800   000075  OFAULT30
       :C                                                                  : 00042900           OFAULT30
       :C          THIS SUBROUTINE COMPUTES MATRIX YB.  SEE REF., EQ.(3)   : 00043000           OFAULT30
       :C                                                                  : 00043100           OFAULT30
0002   :         COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),  : 00043200   000075  YBMAT
0002   :      1                YB(65), YROW(65), GMROW(15), GMCOL(15),     : 00043300   000075  YBMAT
0002   :      2                RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15), : 00043400 000075  YBMAT
0002   :      3                AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM, : 00043500 000075 YBMAT
0002   :      4                IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR     : 00043600   000075  YBMAT
0003   :         INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL, : 00043700   000075  YBMAT
0003   :      1          AROW, ACOL, YBROW, YBCOL                          : 00043800   000075  YBMAT
0004   :         COMPLEX Y, YB, CMPLX                                      : 00043900   000075  YBMAT
       :C                                                                  : 00044000           YBMAT
0005   :         COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50), : 00044100 000075  YBMAT
0005   :      1               Y2(17,17), Y2I(17,17), Y2PI(15,16), A2PLUS(50), : 00044200 000075  YBMAT
0005   :      2               A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP, : 00044300  000075  YBMAT
0005   :      3               N, IB, ITOT, YN(3,3)                         : 00044400   000075  YBMAT
0006   :         INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                    : 00044500   000075  YBMAT
0007   :         INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                    : 00044600   000075  YBMAT
0008   :         COMPLEX YN, Y2, Y2I, Y2PI                                 : 00044700   000075  YBMAT
       :C                                                                  : 00044800           YBMAT
0009   :           COMMON/DELP/  ITYPE,    Y2IP, COSTMN,   JPAR,    NPAR,  ICORR, : 00044900 000075 YBMAT
0009   :    *             IX, IW,    W,    WROW,    WCOL,      X,    XROW,   XCOL : 00045000 000075 YBMAT
0010   :           COMPLEX    Y2IP(17,17), W(80), X(80)                    : 00045100   000075  YBMAT
0011   :           INTEGER    WROW(80), WCOL(80), XROW(80), XCOL(80)       : 00045200   000075  YBMAT
0012   :           INTEGER ITYPE(80), ICORR(80)                            : 00045300   000075  YBMAT
       :C                                                                  : 00045400           YBMAT
       :C        FORM X = -Y*XMU.  ALL MATRICES ARE STORED SPARSELY.       : 00045500           YBMAT
       :C        SINCE Y IS 'DIAGONAL', COMPUTATION IS SIMPLIFIED.         : 00045600           YBMAT
       :C                                                                  : 00045700           YBMAT
0013   :         IF(IXMU .EQ. 0) GO TO 11                                  : 00045800   000075  YBMAT
0015   :         DO 10 L = 1,IXMU                                          : 00045900   000360  YBMAT
0016   :          I1 = XMUROW(L)                                           : 00046000   000451  YBMAT
0017   :          XROW(L) = I1                                             : 00046100   000628  YBMAT
0018   :          XCOL(L) = XMUCOL(L)                                      : 00046200   000803  YBMAT
0019   :          DO 9 K=1,IB                                              : 00046300   001041  YBMAT
0020   :           IF(YROW(K) .EQ. I1) GO TO 10                            : 00046400   001132  YBMAT
0022   : 9        CONTINUE                                                 : 00046500   001340  YBMAT
0023   : 10       X(L) = -Y(K)*XMU(L)                                      : 00046600   001543  YBMAT
0024   : 11      IX = IXMU                                                 : 00046700   002459  YBMAT
       :C                                                                  : 00046800           YBMAT
       :C         FORM W = -Y*RM.  ALL MATRICES ARE STORED SPARSELY.       : 00046900           YBMAT
       :C         SINCE Y IS 'DIAGONAL', COMPUTATION IS SIMPLIFIED.        : 00047000           YBMAT
       :C                                                                  : 00047100           YBMAT
0025   :         IF(IRM .EQ. 0) GO TO 13                                   : 00047200   002567  YBMAT
0027   :         DO 12 L=1,IRM                                             : 00047300   002700  YBMAT
0028   :          I1 = RMROW(L)                                            : 00047400   002791  YBMAT
0029   :          WROW(L) = I1                                             : 00047500   002968  YBMAT
0030   :          WCOL(L) = RMCOL(L)                                       : 00047600   003143  YBMAT
0031   :          DO 14 K=1,IB                                             : 00047700   003381  YBMAT
0032   :           IF(YROW(K) .EQ. I1) GO TO 12                            : 00047800   003472  YBMAT
0034   : 14       CONTINUE                                                 : 00047900   003680  YBMAT
0035   : 12       W(L) = -Y(K)*RM(L)                                       : 00048000   003883  YBMAT
0036   : 13      IW = IRM                                                  : 00048100   004799  YBMAT
       :C                                                                  : 00048200           YBMAT
       :C        FORM X = Y + X + GM.  ALL MATRICES ARE STORED SPARSELY.   : 00048300           YBMAT
       :C        SINCE Y IS 'DIAGONAL', AND X AND GM HAVE NEITHER DIAGONAL : 00048400           YBMAT
       :C      . ELEMENTS NOR ELEMENTS IN COMMON, COMPUTATION IS SIMPLIFIED. : 00048500         YBMAT
       :C                                                                  : 00048600           YBMAT
0037   :         DO 20 L=1,IB                                              : 00048700   004907  YBMAT
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0038  :       X(IX+L) = Y(L)                                        : 00048800   004998  YBMAT
0039  :       XROW(IX+L) = YROW(L)                                  : 00048900   005514  YBMAT
0040  : 20    XCOL(IX+L) = YROW(L)                                  : 00049000   005786  YBMAT
0041  :      IX = IX + IB                                           : 00049100   006202  YBMAT
0042  :      IF(IGM .EQ. 0) GO TO 31                                : 00049200   006350  YBMAT
0044  :      DO 30 L=1,IGM                                          : 00049300   006483  YBMAT
0045  :       X(IX+L) = CMPLX( GM(L), 0.)                           : 00049400   006574  YBMAT
0046  :       XROW(IX+L) = GMROW(L)                                 : 00049500   007030  YBMAT
0047  : 30    XCOL(IX+L) = GMCOL(L)                                 : 00049600   007302  YBMAT
0048  : 31   IX = IX + IGM                                          : 00049700   007713  YBMAT
      :C                                                            : 00049800           YBMAT
      :C      FORM W = 1 + X + ALPHA.  ALL MATRICES ARE STORED SPARSELY.  : 00049900     YBMAT
      :C      SINCE W AND ALPHA HAVE NO COMMON ELEMENTS AND NO DIAGONAL   : 00050000     YBMAT
      :C      ELEMENTS, COMPUTATION IS SIMPLIFIED.                  : 00050100           YBMAT
      :C                                                            : 00050200           YBMAT
0049  :      DO 40 L=1,IB                                           : 00050300   007866  YBMAT
0050  :       W(IW+L) = (1.,0.)                                     : 00050400   007957  YBMAT
0051  :       WROW(IW+L) = L                                        : 00050500   008364  YBMAT
0052  : 40    WCOL(IW+L) = L                                        : 00050600   008573  YBMAT
0053  :      IW = IW + IB                                           : 00050700   008926  YBMAT
0054  :      IF(IALPHA .EQ. 0) GO TO 51                             : 00050800   009074  YBMAT
0056  :      DO 50 L=1,IALPHA                                       : 00050900   009207  YBMAT
0057  :       W(IW+L) = CMPLX( -ALPHA(L), 0.)                       : 00051000   009298  YBMAT
0058  :       WROW(IW+L) = AROW(L)                                  : 00051100   009763  YBMAT
0059  : 50    WCOL(IW+L) = ACOL(L)                                  : 00051200   010035  YBMAT
0060  : 51   IW = IW + IALPHA                                       : 00051300   010451  YBMAT
      :C                                                            : 00051400           YBMAT
      :C       FORM YB = W*X.  ALL MATRICES ARE STORED SPARSELY.    : 00051500           YBMAT
      :C       COMPUTATION IS COMPLICATED BY THE FACT THAT EXISTING : 00051600           YBMAT
      :C       ELEMENTS MUST BE SOUGHT BEFORE THE MULTIPLICATION    : 00051700           YBMAT
      :C       CAN BE PERFORMED.                                    : 00051800           YBMAT
      :C                                                            : 00051900           YBMAT
0061  :      IYB = 0                                                : 00052000   010599  YBMAT
0062  :      DO 90 K=1,IB                                           : 00052100   010690  YBMAT
0063  :       DO 80 L=1,IW                                          : 00052200   010781  YBMAT
0064  :        IF(WCOL(L) - K) 80,55,80                             : 00052300   010872  YBMAT
0065  : 55      DO 70 M=1,IX                                        : 00052400   011116  YBMAT
0066  :          IF(XROW(M) - K) 70,65,70                           : 00052500   011207  YBMAT
0067  : 65       IF(IYB .EQ. 0) GO TO 67                            : 00052600   011451  YBMAT
0069  :           DO 60 NI=1,IYB                                    : 00052700   011584  YBMAT
0070  :            IF(YBROW(NI) - WROW(L)) 60,75,60                 : 00052800   011675  YBMAT
0071  : 75         IF(YBCOL(NI) - XCOL(M)) 60,85,60                 : 00052900   011982  YBMAT
0072  : 60        CONTINUE                                          : 00053000   012289  YBMAT
0073  : 67       IYB = IYB + 1                                      : 00053100   012492  YBMAT
0074  :          YB(IYB) = W(L)*X(M)                                : 00053200   012628  YBMAT
0075  :          YBROW(IYB) = WROW(L)                               : 00053300   013505  YBMAT
0076  :          YBCOL(IYB) = XCOL(M)                               : 00053400   013743  YBMAT
0077  :          GO TO 70                                           : 00053500   013981  YBMAT
0078  : 85       YB(NI) = YB(NI) + W(L)*X(M)                        : 00053600   014067  YBMAT
0079  : 70      CONTINUE                                            : 00053700   015213  YBMAT
0080  : 80     CONTINUE                                             : 00053800   015416  YBMAT
0081  : 90    CONTINUE                                              : 00053900   015619  YBMAT
0082  :      RETURN                                                 : 00054000   015822  YBMAT
0083  :      END                                                    : 00054100   015936  YBMAT

NO ERRORS AND NO WARNINGS IN 83 STATEMENTS,CODE EMITTED = 15995 BITS (2000 BYTES)YBMAT
COMPILE TIME IS 46.1 SECONDS FOR 114 CARDS AT 148 CARDS/MINUTE.
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BURROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,        TUESDAY   10/02/79    05:18 PM
     /(E99000)  /FAULT30
 0001    :        SUBROUTINE ANEW                                              : 00054200    000075   OFAULT30
         :C                                                                    : 00054300             OFAULT30
         :C          THIS SUBROUTINE COMPUTES THE NODE-TO-BRANCH INCIDENCE MATRICES : 00054400        OFAULT30
         :C          (SPARSELY STORED VECTORS A1PLSP, A1MNSP, A2PLSP, A2MNSP) FOR   : 00054500        OFAULT30
         :C          THE NETWORK WITH NODES I AND J SHORTED.  SEE REF., EQ.(13).     : 00054600        OFAULT30
         :C                                                                    : 00054700             OFAULT30
 0002    :        COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50), : 00054800    000075   ANEW
 0002    :       1             Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50), : 00054900    000075   ANEW
 0002    :       2             A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,   : 00055000    000075   ANEW
 0002    :       3             N, IB, ITOT, YN(3,3)                            : 00055100    000075   ANEW
 0003    :        INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                       : 00055200    000075   ANEW
 0004    :        INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                       : 00055300    000075   ANEW
 0005    :        COMPLEX YN, Y2, Y2I, Y2PI                                    : 00055400    000075   ANEW
 0006    :        DO 150 L=1,IB                                                : 00055500    000075   ANEW
 0007    :        A1PLSP(L) = A1PLUS(L)                                        : 00055600    000318   ANEW
 0008    :        A1MNSP(L) = A1MNUS(L)                                        : 00055700    000556   ANEW
 0009    :        IF(A2PLUS(L) - (J-IP)) 20,30,40                              : 00055800    000794   ANEW
 0010    : 20     A2PLSP(L) = A2PLUS(L)                                        : 00055900    001072   ANEW
 0011    :        GO TO 90                                                     : 00056000    001310   ANEW
 0012    : 30     IF(I-IP) 50,50,60                                            : 00056100    001396   ANEW
 0013    : 50     IF(I .EQ. 0) GO TO 55                                        : 00056200    001577   ANEW
 0015    :        IF(A1MNSP(L) - I) 52,51,52                                   : 00056300    001710   ANEW
 0016    : 51     A1MNSP(L) = 0                                                : 00056400    001954   ANEW
 0017    :        GO TO 55                                                     : 00056500    002117   ANEW
 0018    : 52     A1PLSP(L) = I                                                : 00056600    002203   ANEW
 0019    : 55     A2PLSP(L)  = 0                                               : 00056700    002378   ANEW
 0020    :        GO TO 90                                                     : 00056800    002541   ANEW
 0021    : 60     IF(A2MNUS(L) - (I-IP)) 80,70,80                              : 00056900    002627   ANEW
 0022    : 70     A2PLSP(L) = 0                                                : 00057000    002905   ANEW
 0023    :        A2MNSP(L) = 0                                                : 00057100    003069   ANEW
 0024    :        GO TO 90                                                     : 00057200    003231   ANEW
 0025    : 80     A2PLSP(L) = I-IP                                             : 00057300    003317   ANEW
 0026    :        GO TO 90                                                     : 00057400    003526   ANEW
 0027    : 40     A2PLSP(L) = A2PLUS(L) - 1                                    : 00057500    003612   ANEW
 0028    : 90     IF(A2MNUS(L) - (J-IP)) 100,110,120                           : 00057600    003872   ANEW
 0029    : 100    A2MNSP(L) = A2MNUS(L)                                        : 00057700    004150   ANEW
 0030    :        GO TO 150                                                    : 00057800    004388   ANEW
 0031    : 110    IF(I-IP) 130,130,135                                         : 00057900    004474   ANEW
 0032    : 130    IF(I .EQ. 0) GO TO 134                                       : 00058000    004655   ANEW
 0034    :        IF(A1PLSP(L) - I) 132,131,132                                : 00058100    004788   ANEW
 0035    : 131    A1PLSP(L) = 0                                                : 00058200    005032   ANEW
 0036    : 132    A1MNSP(L) = I                                                : 00058300    005195   ANEW
 0037    : 134    A2MNSP(L) = 0                                                : 00058400    005370   ANEW
 0038    :        GO TO 150                                                    : 00058500    005533   ANEW
 0039    : 135    IF(A2PLUS(L) - (I-IP)) 145,140,145                           : 00058600    005619   ANEW
 0040    : 140    A2PLSP(L) = 0                                                : 00058700    005897   ANEW
 0041    :        A2MNSP(L) = 0                                                : 00058800    006060   ANEW
 0042    :        GO TO 150                                                    : 00058900    006223   ANEW
 0043    : 145    A2MNSP(L) = I-IP                                             : 00059000    006309   ANEW
 0044    :        GO TO 150                                                    : 00059100    006518   ANEW
 0045    : 120    A2MNSP(L) = A2MNUS(L) - 1                                    : 00059200    006604   ANEW
 0046    : 150    CONTINUE                                                     : 00059300    006864   ANEW
 0047    :        RETURN                                                       : 00059400    007067   ANEW
 0048    :        END                                                          : 00059500    007181   ANEW

NO ERRORS AND NO WARNINGS IN 48 STATEMENTS,CODE EMITTED = 7240 BITS (905 BYTES)ANEW
COMPILE TIME IS 35.3 SECONDS FOR 54 CARDS AT 92 CARDS/MINUTE.
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BURROUGHS B1300/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY   10/02/79   05:18 PM
      /(E99000)  /FAULT30
 0001   :      SUBROUTINE YMAT(A1PLSP,A1MNSP,A2PLSP,A2MNSP,Y2PI,YN,IPP,IB,IQT)   : 00059600  000075  OFAULT30
        :C                                                                       : 00059700          OFAULT30
        :C       THIS SUBROUTINE COMPUTES THE Y-PARAMETERS AT THE ACCESSIBLE     : 00059800          OFAULT30
        :C       NODES.  SEE REF., EQ.(8).  SPARSE MATRIX TECHNIQUES ARE         : 00059900          OFAULT30
        :C       USED, AS DESCRIBED IN REF., PP. 18-22.                          : 00060000          OFAULT30
        :C                                                                       : 00060100          OFAULT30
 0002   :      COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),          : 00060200  000075  YMAT
 0002   :     1              YB(65), YROW(65), GMROW(15), GMCOL(15),             : 00060300  000075  YMAT
 0002   :     2              RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),       : 00060400  000075  YMAT
 0002   :     3              AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,      : 00060500  000075  YMAT
 0002   :     4              IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR             : 00060600  000075  YMAT
 0003   :      INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,         : 00060700  000075  YMAT
 0003   :     1        AROW, ACOL, YBROW, YBCOL                                  : 00060800  000075  YMAT
 0004   :      COMPLEX Y, YB                                                     : 00060900  000075  YMAT
 0005   :      COMPLEX Y2PI(IQT,IQT),YN(3,3)                                     : 00061000  000075  YMAT
 0006   :      INTEGER A2PLSP(50),A2MNSP(50)                                     : 00061100  000075  YMAT
 0007   :      INTEGER A1PLSP(50),A1MNSP(50)                                     : 00061200  000075  YMAT
        :C                                                                       : 00061220          YMAT
 0008   :      COMMON/DEL/ C,D                                                   : 00061240  000075  YMAT
        :C                                                                       : 00061260          YMAT
 0009   :      COMPLEX W,C(50,50),D(50,50),E(50,50)                              : 00061300  000075  YMAT
        :C                                                                       : 00061400          YMAT
 0010   :      DO 100 I=1,IB                                                     : 00061500  000075  YMAT
 0011   :        I1 = A2PLSP(I)                                                  : 00061600  000421  YMAT
 0012   :        I2 = A2MNSP(I)                                                  : 00061700  000584  YMAT
 0013   :      DO 100 J=1,IB                                                     : 00061800  000747  YMAT
 0014   :          D(I,J) = (0.,0.)                                              : 00061900  000838  YMAT
 0015   :          E(I,J) = (0.,0.)                                              : 00062000  001263  YMAT
 0016   :        J1 = A2PLSP(J)                                                  : 00062100  001688  YMAT
 0017   :        J2 = A2MNSP(J)                                                  : 00062200  001851  YMAT
 0018   :        W = (0.,0.)                                                     : 00062300  002014  YMAT
 0019   :        IF(I1*J1 .NE. 0) W = W + Y2PI(I1,J1)                            : 00062400  002211  YMAT
 0021   :        IF(I2*J2 .NE. 0) W = W + Y2PI(I2,J2)                            : 00062500  002809  YMAT
 0023   :        IF(I1*J2 .NE. 0) W = W - Y2PI(I1,J2)                            : 00062600  003407  YMAT
 0025   :        IF(I2*J1 .NE. 0) W = W - Y2PI(I2,J1)                            : 00062700  004069  YMAT
 0027   : 100    C(I,J) = W                                                      : 00062800  004731  YMAT
        :C                                                                       : 00062900          YMAT
 0028   :      DO 5 I=1,IB                                                       : 00063000  005416  YMAT
 0029   :      DO 5 L=1,IYB                                                      : 00063100  005507  YMAT
 0030   :       K = YBROW(L)                                                     : 00063200  005598  YMAT
 0031   :       J = YBCOL(L)                                                     : 00063300  005775  YMAT
 0032   : 5    D(I,J) = D(I,J) - C(I,K)*YB(L)                                    : 00063400  005952  YMAT
        :C                                                                       : 00063500          YMAT
 0033   :      DO 10 K=1,IB                                                      : 00063600  007542  YMAT
 0034   : 10   D(K,K) = D(K,K) + (1.,0.)                                         : 00063700  007633  YMAT
        :C                                                                       : 00064000          YMAT
 0035   :      DO 15 J=1,IB                                                      : 00064100  008514  YMAT
 0036   :      DO 15 L=1,IYB                                                     : 00064200  008605  YMAT
 0037   :       I = YBROW(L)                                                     : 00064300  008696  YMAT
 0038   :       K = YBCOL(L)                                                     : 00064400  008873  YMAT
 0039   : 15   E(I,J) = E(I,J) + YB(L)*D(K,J)                                    : 00064500  009050  YMAT
        :C                                                                       : 00064600          YMAT
 0040   :      DO 20 I=1,IPP                                                     : 00064700  010640  YMAT
 0041   :      DO 20 J=1,IPP                                                     : 00064800  010731  YMAT
 0042   : 20   YN(I,J) = (0.,0.)                                                 : 00064900  010822  YMAT
        :C                                                                       : 00065000          YMAT
        :C      COMPUTE A1 * E * A1T.  A1 IS STORED SPARSELY.                    : 00065100          YMAT
        :C                                                                       : 00065200          YMAT
 0043   :      DO 30 L=1,IB                                                      : 00065300  011485  YMAT
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0044    :          IP = A1PLSP(L)                                          : 00065400    011576  YMAT
0045    :          IM = A1MNSP(L)                                          : 00065500    011739  YMAT
0046    :       DO 30 K=1,IB                                               : 00065600    011902  YMAT
0047    :          JP = A1PLSP(K)                                          : 00065700    011993  YMAT
0048    :          JM = A1MNSP(K)                                          : 00065800    012156  YMAT
0049    :          IF(IP*JP .NE. 0) YN(IP,JP) = YN(IP,JP) + E(L,K)         : 00065900    012319  YMAT
0051    :          IF(IM*JM .NE. 0) YN(IM,JM) = YN(IM,JM) + E(L,K)         : 00066000    013348  YMAT
0053    :          IF(IP*JM .NE. 0) YN(IP,JM) = YN(IP,JM) - E(L,K)         : 00066100    014377  YMAT
0055    :          IF(IM*JP .NE. 0) YN(IM,JP) = YN(IM,JP) - E(L,K)         : 00066200    015406  YMAT
0057    :  30   CONTINUE                                                   : 00066300    016435  YMAT
0058    :       RETURN                                                     : 00066400    016782  YMAT
0059    :       END                                                        : 00066500    016896  YMAT

ERRORS AND NO WARNINGS IN 59 STATEMENTS,CODE EMITTED = 16955 BITS (2120 BYTES)YMAT
PILE TIME IS 40.5 SECONDS FOR 71 CARDS AT 105 CARDS/MINUTE.
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IRROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79    05:18 PM
      /(E99C00)   /FAULT30
 0001    :       SUBROUTINE Y2PMAT(Y2P,IQP,IQPT,A2PLSP,A2MNSP)                   : 00066600   000075  OFAULT30
         :C                                                                      : 00066700           OFAULT30
         :C        THIS SUBROUTINE FORMS EITHER Y2 = A2 * YB * A2T OR            : 00066800           OFAULT30
         :C        Y2P = A2P * YB * A2PT.   SEE REF., EQ.(2).   SPARSE MATRIX    : 00066900           OFAULT30
         :C        TECHNIQUES HAVE BEEN USED TO STORE YB.                        : 00067000           OFAULT30
         :C                                                                      : 00067100           OFAULT30
 0002    :       COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),        : 00067200   000075  Y2PMAT
 0002    :      1               YB(65), YROW(65), GMROW(15), GMCOL(15),          : 00067300   000075  Y2PMAT
 0002    :      2               RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),    : 00067400   000075  Y2PMAT
 0002    :      3               AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,   : 00067500   000075  Y2PMAT
 0002    :      4               IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR          : 00067600   000075  Y2PMAT
 0003    :       INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,        : 00067700   000075  Y2PMAT
 0003    :      1        AROW, ACOL, YBROW, YBCOL                                 : 00067800   000075  Y2PMAT
 0004    :       COMPLEX Y, YB                                                   : 00067900   000075  Y2PMAT
 0005    :       COMPLEX Y2P(IQPT,IQPT)                                          : 00068000   000075  Y2PMAT
 0006    :       INTEGER A2PLSP(50), A2MNSP(50)                                  : 00068100   000075  Y2PMAT
         :C                                                                      : 00068200           Y2PMAT
         :C        INITIALIZE Y2P TO 0.                                          : 00068300           Y2PMAT
         :C                                                                      : 00068400           Y2PMAT
 0007    :       DO 10 I=1,IQP                                                   : 00068500   000075  Y2PMAT
 0008    :       DO 10 J=1,IQP                                                   : 00068600   000401  Y2PMAT
 0009    : 10    Y2P(I,J) = (0.,0.)                                              : 00068700   000492  Y2PMAT
         :C                                                                      : 00068800           Y2PMAT
         :C        COMPUTE Y2P = A2 * YB * A2T                                   : 00068900           Y2PMAT
         :C                                                                      : 00069000           Y2PMAT
 0010    :       DO 100 M=1,IYB                                                  : 00069100   001155  Y2PMAT
 0011    :        L = YBROW(M)                                                   : 00069200   001246  Y2PMAT
 0012    :        K = YBCOL(M)                                                   : 00069300   001423  Y2PMAT
 0013    :        IP  = A2PLSP(L)                                                : 00069400   001600  Y2PMAT
 0014    :        IM = A2MNSP(L)                                                 : 00069500   001763  Y2PMAT
 0015    :        JP  = A2PLSP(K)                                                : 00069600   001926  Y2PMAT
 0016    :        JM = A2MNSP(K)                                                 : 00069700   002089  Y2PMAT
 0017    :        IF(IP*JP .NE. 0) Y2P(IP,JP) = Y2P(IP,JP) + YB(M)               : 00069800   002252  Y2PMAT
 0019    :        IF(IM*JM .NE. 0) Y2P(IM,JM) = Y2P(IM,JM) + YB(M)               : 00069900   003229  Y2PMAT
 0021    :        IF(IP*JM .NE. 0) Y2P(IP,JM) = Y2P(IP,JM) - YB(M)               : 00070000   004206  Y2PMAT
 0023    :        IF(IM*JP .NE. 0) Y2P(IM,JP) = Y2P(IM,JP) - YB(M)               : 00070100   005183  Y2PMAT
 0025    : 100   CONTINUE                                                        : 00070200   006160  Y2PMAT
 0026    :       RETURN                                                          : 00070300   006363  Y2PMAT
 0027    :       END                                                             : 00070400   006477  Y2PMAT

) ERRORS AND NO WARNINGS IN 27 STATEMENTS,CODE EMITTED = 6536 BITS (817 BYTES)Y2PMAT
)MPILE TIME IS 31.3 SECONDS FOR 39 CARDS AT 75 CARDS/MINUTE.
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0001    :        SUBROUTINE SNEW                                                          : 00070500   000075   OFAULT30
        :C                                                                                : 00070600            OFAULT30
        :C         THIS SUBROUTINE FORMS THE INVERSE OF Y2P, CALLED Y2PI,                 : 00070700            OFAULT30
        :C         WITH NODES I AND J SHORTED, USING EQ.(20) IN THE REF.                  : 00070800            OFAULT30
        :C         SPARSE MATRIX TECHNIQUES ARE USED.  SEE REF., PP. 14-18.               : 00070900            OFAULT30
        :C                                                                                : 00071000            OFAULT30
0002    :        COMPLEX YN, Y2, Y2I, Y2PI                                                : 00071100   000075   SNEW
0003    :        COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),             : 00071200   000075   SNEW
0003    :       1             Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),             : 00071300   000075   SNEW
0003    :       2             A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,               : 00071400   000075   SNEW
0003    :       3             N, IB, ITOT, YN(3,3)                                        : 00071500   000075   SNEW
0004    :        INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                                   : 00071600   000075   SNEW
0005    :        INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                                   : 00071700   000075   SNEW
0006    :        COMPLEX XK(16,16),XM(16),C(16),XI(16,16),W,MC                            : 00071800   000075   SNEW
0007    :        INTEGER V(17), XIROW(16)                                                 : 00071900   000075   SNEW
        :C                                                                                : 00072000            SNEW
        :C         FORM VECTOR V.  SEE REF., P. 17                                        : 00072100            SNEW
        :C                                                                                : 00072200            SNEW
0008    :        DO 10 K=1,17                                                             : 00072300   000075   SNEW
0009    : 10     V(K) = 0                                                                 : 00072400   000318   SNEW
0010    :        IF(J .EQ. IP+1) GO TO 25                                                 : 00072500   000613   SNEW
0012    :        DO 20 K=1,J-IP-1                                                         : 00072600   000866   SNEW
0013    : 20     V(K) = K                                                                 : 00072700   001164   SNEW
0014    : 25     IQ = N-IP                                                                : 00072800   001386   SNEW
0015    :        DO 30 K = J-IP,IQ-1                                                      : 00072900   001534   SNEW
0016    : 30     V(K) = K+1                                                               : 00073000   001875   SNEW
0017    :        V(IQ) = J-IP                                                             : 00073100   002099   SNEW
        :C                                                                                : 00073200            SNEW
        :C         FORM MATRICES K AND M, DEFINED IN REF., EQ.(22).  SPARSE               : 00073300            SNEW
        :C         MATRIX TECHNIQUES ARE USED.  SEE REF., EQ.(24) AND (25).               : 00073400            SNEW
        :C                                                                                : 00073500            SNEW
0018    :        DO 50 IR=1,IQ                                                            : 00073600   002308   SNEW
0019    :        DO 50 IS=1,IQ-1                                                          : 00073700   002399   SNEW
0020    :        W = Y2I(V(IR),V(IS))                                                     : 00073800   002683   SNEW
0021    :        IF(IR-IQ) 60,65,65                                                       : 00073900   003167   SNEW
0022    : 60     XK(IR,IS) = W                                                            : 00074000   003348   SNEW
0023    :        GO TO 50                                                                 : 00074100   003745   SNEW
0024    : 65     XM(IS) = W-Y2I(I-IP,V(IS))                                               : 00074200   003831   SNEW
0025    : 50     CONTINUE                                                                 : 00074300   004649   SNEW
        :C                                                                                : 00074400            SNEW
        :C         FORM R22 * Y2 * U.  SEE REF., EQ.(26).                                 : 00074500            SNEW
        :C                                                                                : 00074600            SNEW
0026    :        DO 70 M=1,IQ-1                                                           : 00074700   004879   SNEW
0027    : 70     C(M) = Y2(V(M),J-IP)                                                     : 00074800   005163   SNEW
0028    :        IF(I .GT. IP) C(I-IP) = C(I-IP) + Y2(J-IP,J-IP)                          : 00074900   005821   SNEW
        :C                                                                                : 00075000            SNEW
        :C         FORM 1 - M * (R22 * Y2 * U).  SEE REF., EQ.(21).                       : 00075100            SNEW
        :C                                                                                : 00075200            SNEW
0030    :        MC = (1.,0.)                                                             : 00075300   006960   SNEW
0031    :        DO 80 M=1,IQ-1                                                           : 00075400   007157   SNEW
0032    :        IF(REAL(C(M)) .NE. 0. .AND. AIMAG(C(M)) .NE. 0.) MC=MC-XM(M)*C(M)        : 00075500   007441   SNEW
0034    : 80     CONTINUE                                                                 : 00075600   008202   SNEW
        :C                                                                                : 00075700            SNEW
        :C         FORM XI = C * XM / MC.  SINCE C HAS MANY ZERO ELEMENTS,                : 00075800            SNEW
        :C         XI HAS MANY ZERO ROWS.  ONLY THE NON-ZERO ROWS OF XI ARE               : 00075900            SNEW
        :C         FORMED BELOW.  SEE REF., EQ. (21).                                     : 00076000            SNEW
        :C                                                                                : 00076100            SNEW
0035    :        IROW = 0                                                                 : 00076200   008988   SNEW
0036    :        DO 85 K=1,IQ-1                                                           : 00076300   009079   SNEW
0037    :         IF(REAL(C(K)) .EQ. 0. .AND. AIMAG(C(K)) .EQ. 0.) GO TO 85               : 00076400   009363   SNEW
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0039  :       IROW = IROW + 1                                    : 00076500   010077  SNEW
0040  :       XIROW(IROW) = K                                    : 00076600   010213  SNEW
0041  :       DO 85 M=1,IQ-1                                     : 00076700   010388  SNEW
0042  :       XI(IROW,M) = C(K)*XM(M)/MC                         : 00076800   010672  SNEW
0043  : 85   CONTINUE                                            : 00076900   011749  SNEW
      :C                                                         : 00077000           SNEW
      :C      FORM Y2PI = XK * (XI + 1).   RECALL THAT ONLY THE NON-  : 00077100      SNEW
      :C      ZERO ROWS OF XI HAVE BEEN SAVED.  SEE REF., EQ.(21).    : 00077200      SNEW
      :C                                                         : 00077300           SNEW
0044  :      DO 95 I1=1,IQ-1                                     : 00077400   011862  SNEW
0045  :      DO 95 J1=1,IQ-1                                     : 00077500   012146  SNEW
0046  :       W = (0.,0.)                                        : 00077600   012430  SNEW
0047  :       DO 90 L=1,IROW                                     : 00077700   012627  SNEW
0048  :        K = XIROW(L)                                      : 00077800   012718  SNEW
0049  : 90    W = W + XK(I1,K)*XI(L,J1)                          : 00077900   012895  SNEW
0050  : 95   Y2PI(I1,J1) = W + XK(I1,J1)                         : 00078000   013912  SNEW
0051  :      RETURN                                              : 00078100   014675  SNEW
0052  :      END                                                 : 00078200   014789  SNEW

0 ERRORS AND NO WARNINGS IN 52 STATEMENTS,CODE EMITTED = 14848 BITS (1856 BYTES)SNEW
COMPILE TIME IS 39.0 SECONDS FOR 78 CARDS AT 120 CARDS/MINUTE.
```

```
URROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY   10/02/79    05:18 PM
        /(F99000)  /FAULT30
 0001    :       SUBROUTINE MINVC(B,C,NN,NT)                                       : 00078300   000075  OFAULT30
         :C                                                                        : 00078400           OFAULT30
         :C        THIS SUBROUTINE INVERTS THE COMPLEX VALUED MATRIX B.            : 00078500           OFAULT30
         :C        IN THE CALLING PROGRAM, B HAS BEEN DIMENSIONED NT X NT.         : 00078600           OFAULT30
         :C        THE ACTUAL DIMENSION USED IN THE CALLING PROGRAM IS NN X NN.    : 00078700           OFAULT30
         :C        THE INVERSE OF B IS RETURNED AS C, WHICH IS DIMENSIONED         : 00078800           OFAULT30
         :C        AS B.                                                           : 00078900           OFAULT30
         :C                                                                        : 00079000           OFAULT30
 0002    :       COMPLEX ALPHA(17),BETA(17),SUM,SUMP,U(17),V(17),LAMBDA            : 00079100   000075  MINVC
 0003    :       COMPLEX A(17,17),B(NT,NT),C(NT,NT)                                : 00079200   000075  MINVC
 0004    :       DO 1 I=1,NN                                                       : 00079300   000075  MINVC
 0005    :       DO 1 J=1,NN                                                       : 00079400   000451  MINVC
 0006    :       C(I,J) = (0.,0)                                                   : 00079500   000542  MINVC
 0007    :     1 A(I,J) = B(I,J)                                                   : 00079600   000917  MINVC
 0008    :       DO 3 I=1,NN                                                       : 00079700   001741  MINVC
 0009    :       A(I,I) = A(I,I) - (1.,0.)                                         : 00079800   001832  MINVC
 0010    :     3 C(I,I) = (1.,0.)                                                  : 00079900   002569  MINVC
 0011    :       DO 40 N = 1,NN                                                    : 00080000   003088  MINVC
 0012    :       DO 10 I = N+1,NN                                                  : 00080100   003179  MINVC
 0013    :       ALPHA(I) = A(I,N)/A(N,N)                                          : 00080200   003315  MINVC
 0014    :    10 BETA(I) = A(N,I)                                                  : 00080300   004296  MINVC
 0015    :       DO 17 I=1,NN                                                      : 00080400   004974  MINVC
 0016    :       SUM = (0.,0.)                                                     : 00080500   005065  MINVC
 0017    :       SUMP = (0.,0.)                                                    : 00080600   005262  MINVC
 0018    :       DO 15 K=N,NN                                                      : 00080700   005459  MINVC
 0019    :       SUM = SUM + C(I,K)*A(K,N)                                         : 00080800   005556  MINVC
 0020    :    15 SUMP = SUMP + A(N,K)*C(K,I)                                       : 00080900   006379  MINVC
 0021    :       U(I) = SUM                                                        : 00081000   007346  MINVC
 0022    :    17 V(I) = SUMP                                                       : 00081100   007691  MINVC
 0023    :       SUM = (0.,0.)                                                     : 00081200   008180  MINVC
 0024    :       DO 21 J = N,NN                                                    : 00081300   008377  MINVC
 0025    :    21 SUM = SUM + V(J)*A(J,N)                                           : 00081400   008474  MINVC
 0026    :       LAMBDA = SUM + A(N,N)                                             : 00081500   009439  MINVC
 0027    :       DO 30 I = 1,NN                                                    : 00081600   009920  MINVC
 0028    :       DO 30 J = 1,NN                                                    : 00081700   010011  MINVC
 0029    :    30 C(I,J) = C(I,J) - U(I)*V(J)/LAMBDA                                : 00081800   010102  MINVC
 0030    :       IF(N .EQ. NN) RETURN                                              : 00081900   011688  MINVC
 0032    :       DO 38 I = N+1,NN                                                  : 00082000   011947  MINVC
 0033    :       DO 38 J = N+1,NN                                                  : 00082100   012083  MINVC
 0034    :    38 A(I,J) = A(I,J) - ALPHA(I)*BETA(J)                                : 00082200   012219  MINVC
 0035    :    40 CONTINUE                                                          : 00082300   013757  MINVC
 0036    :       END                                                               : 00082400   013960  MINVC
 ERRORS AND NO WARNINGS IN 36 STATEMENTS,CODE EMITTED = 14074 BITS (1760 BYTES)MINVC
MPILE TIME IS 31.3 SECONDS FOR 42 CARDS AT 81 CARDS/MINUTE.
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```
JRROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY   10/02/79   05:18 PM
      /(E99000)   /FAULT30
  0001   :        FUNCTION COSTF(YNNOM)                                        : 00082500   000075  OFAULT30
  0002   :        COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50), : 00082600   000075  COSTF
  0002   :      1               Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50), : 00082700   000075  COSTF
  0002   :      2               A2MNUS(50), A2PLSP(50), A2MNSP(50), I1, J1, IP, : 00082800   000075  COSTF
  0002   :      3               N, IB, ITOT, YN(3,3)                          : 00082900   000075  COSTF
  0003   :        INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                       : 00083000   000075  COSTF
  0004   :        INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                       : 00083100   000075  COSTF
  0005   :        COMPLEX YN, Y2, Y2I, Y2PI                                    : 00083200   000075  COSTF
  0006   :        COMPLEX YNNOM(3,3)                                           : 00083300   000075  COSTF
  0007   :        DIMENSION DENOM(3,3)                                         : 00083400   000075  COSTF
  0008   :        COSTF = 0.                                                   : 00083500   000075  COSTF
  0009   :        DO 10 I = 1,IP                                               : 00083600   000411  COSTF
  0010   :        DO 10 J = 1,IP                                               : 00083700   000502  COSTF
  0011   :          D = CABS(YNNOM(I,J))                                       : 00083800   000593  COSTF
  0012   :          IF(D .EQ. 0.) D = 1.E-9                                    : 00083900   000990  COSTF
  0014   :    10  DENOM(I,J) = D                                               : 00084000   001277  COSTF
  0015   :        DO 20 I = 1,IP                                               : 00084100   001792  COSTF
  0016   :        DO 20 J = 1,IP                                               : 00084200   001883  COSTF
  0017   :    20  COSTF = COSTF + CABS(YN(I,J) - YNNOM(I,J))/DENOM(I,J)        : 00084300   001974  COSTF
  0018   :        RETURN                                                       : 00084400   003218  COSTF
  0019   :        END                                                          : 00084500   003332  COSTF

C ERRORS AND NO WARNINGS IN 19 STATEMENTS,CODE EMITTED = 3391 BITS (424 BYTES)COSTF
OMPILE TIME IS 25.2 SECONDS FOR 21 CARDS AT 50 CARDS/MINUTE.
```

```
ROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79    05:18 PM
  /(E99000)  /FAULT30
 :         SUBROUTINE SEARCH(A, B, H, K, X, Y)                                 : 00084600   000075   OFAULT30
 :         LOGICAL K                                                           : 00084700   000075   SEARCH
 :         REAL L1, L2                                                         : 00084800   000075   SEARCH
 :C                                                                            : 00084900            SEARCH
 :C        THIS SUBROUTINE PERFORMS A FIBONACI SEARCH FOR A MINIMUM            : 00085000            SEARCH
 :C        OF THE FUNCTION F(.) OVER THE INTERVAL (A,B).  THE FINAL            : 00085100            SEARCH
 :C        INTERVAL IS NO LARGER THAN H.  Y(X) IS THE APPROXIMATE              : 00085200            SEARCH
 :C        MAXIMUM OF THE FUNCTION.                                            : 00085300            SEARCH
 :C                                                                            : 00085400            SEARCH
 :         C = B - A                                                           : 00085500   000075   SEARCH
 :         FN = C/H                                                            : 00085600   000399   SEARCH
 :         G1 = 1.                                                             : 00085700   000547   SEARCH
 :         G2 = 1.                                                             : 00085800   000675   SEARCH
 :         DO 10 I=2,25                                                        : 00085900   000803   SEARCH
 :           G3 = G1 + G2                                                      : 00086000   000898   SEARCH
 :           G = G2                                                            : 00086100   001046   SEARCH
 :           G2 = G3                                                           : 00086200   001143   SEARCH
 :           G1 = G                                                            : 00086300   001240   SEARCH
 :           IF(G2 .GE. FN) GO TO 20                                           : 00086400   001337   SEARCH
 :    10   CONTINUE                                                            : 00086500   001482   SEARCH
 :    20   L2 = G1/G2                                                          : 00086600   001673   SEARCH
 :         L1 = 1.                                                             : 00086700   001821   SEARCH
 :         IF(K) WRITE(3,100)                                                  : 00086800   001949   SEARCH
 :         X = A + L2*C                                                        : 00086900   002333   SEARCH
 :         X1 = X                                                              : 00087000   002515   SEARCH
 :         X2 = X                                                              : 00087100   002623   SEARCH
 :         F1 = F(X1)                                                          : 00087200   002731   SEARCH
 :         DO 50 N = 2,I-1                                                     : 00087300   002902   SEARCH
 :           IF(A - X2) 30, 40, 40                                             : 00087320   003190   SEARCH
 :    30       X = B - L2*C                                                    : 00087340   003371   SEARCH
 :             X1 = X2                                                         : 00087360   003553   SEARCH
 :             X2 = X                                                          : 00087380   003650   SEARCH
 :             F2 = F(X2)                                                      : 00087400   003758   SEARCH
 :           GO TO 45                                                          : 00087420   003929   SEARCH
 :    40       X = A + L2*C                                                    : 00087440   004015   SEARCH
 :             X2 = X1                                                         : 00087460   004197   SEARCH
 :             X1 = X                                                          : 00087480   004294   SEARCH
 :             F1 = F(X1)                                                      : 00087500   004402   SEARCH
 :    45     IF(K) WRITE(3,101) X1, X2, F1, F2                                 : 00087520   004573   SEARCH
 :           IF(F1 - F2) 46, 47, 47                                            : 00087540   005384   SEARCH
 :    46       B = X1                                                          : 00087560   005565   SEARCH
 :             F1 = F2                                                         : 00087580   005673   SEARCH
 :           GO TO 48                                                          : 00087600   005770   SEARCH
 :    47       A = X2                                                          : 00087620   005856   SEARCH
 :             F2 = F1                                                         : 00087640   005964   SEARCH
 :    48     Z = L2                                                            : 00087660   006061   SEARCH
 :           L2 = L1 - L2                                                      : 00087680   006158   SEARCH
 :           L1 = Z                                                            : 00087700   006306   SEARCH
 :    50   CONTINUE                                                            : 00087720   006403   SEARCH
 :         X = (A + B)/2.                                                      : 00087740   006489   SEARCH
 :         Y = F(X)                                                            : 00087760   006685   SEARCH
 :         WRITE(3,102) X, Y, A, B, I                                          : 00087780   006856   SEARCH
 :         RETURN                                                              : 00087800   007690   SEARCH
 :   100   FORMAT(//, 10X, 'X1', 10X, 'X2', 10X, 'F1', 10X, 'F2'/)            : 00087820   007804   SEARCH
 :   101   FORMAT(4E12.3)                                                      : 00087840   007804   SEARCH
 :   102   FORMAT(/5X, 'X =', E12.4, 5X, 'Y =', E12.4/5X,                      : 00087860   007804   SEARCH
 :     *     'MAXIMUM LIES BETWEEN', E12.4, ' AND', E12.4,                     : 00087880   007804   SEARCH
 :     *.    5X, '(', I2, ' FUNCTION EVALUATIONS)'/ )                          : 00087890   007804   SEARCH
 :         END                                                                 : 00087900   007804   SEARCH
```

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RROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          MONDAY     10/08/79     01:43 PM
      /(E99000)    /FAULTO
 0001   :              SUBROUTINE UPDATE(PAR)                                   : 00088800   000075  OFAULTO
 0002   :              COMMON/DELP/  ITYPE,    Y2IP, COSTMN,   JPAR,   NPAR,  ICORR,  : 00088900   000075  UPDATE
 0002   :        *                W,    WROW,    WCOL,      X,    XROW,    XCOL    : 00089000   000075  UPDATE
 0003   :              COMPLEX    Y2IP(17,17), W(80), X(80)                      : 00089100   000075  UPDATE
 0004   :              INTEGER    WROW(80), WCOL(80), XROW(80), XCOL(80)         : 00089200   000075  UPDATE
 0005   :              INTEGER ITYPE(80), ICORR(80)                              : 00089300   000075  UPDATE
 0006   :              COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),  : 00089400   000075  UPDATE
 0006   :        1                YB(65), YROW(65), GMROW(15), GMCOL(15),        : 00089500   000075  UPDATE
 0006   :        2               RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),   : 00089600   000075  UPDATE
 0006   :        3               AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,  : 00089700   000075  UPDATE
 0006   :        4               IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR         : 00089800   000075  UPDATE
 0007   :              INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL, : 00089900   000075  UPDATE
 0007   :        1          AROW, ACOL, YBROW, YBCOL                             : 00090000   000075  UPDATE
 0008   :              COMPLEX Y, YB                                             : 00090100   000075  UPDATE
 0009   :              COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50), : 00090200   000075  UPDATE
 0009   :        1              Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50), : 00090300   000075  UPDATE
 0009   :        2              A2MNUS(50), A2PLSP(50), A2MNSP(50), I1, J1, IP,  : 00090400   000075  UPDATE
 0009   :        3              N, IB, ITOT, YN(3,3)                             : 00090500   000075  UPDATE
 0010   :              INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                    : 00090600   000075  UPDATE
 0011   :              INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                    : 00090700   000075  UPDATE
 0012   :              COMPLEX YN, Y2, Y2I, Y2PI, CMPLX                          : 00090800   000075  UPDATE
        :C                                                                       : 00090900           UPDATE
 0013   :              GO TO (1,2,3,4,5), JPAR                                   : 00091000   000075  UPDATE
 0014   :        1     XT = REAL( Y(NPAR) )                                      : 00091100   000422  UPDATE
 0015   :              YT = AIMAG( Y(NPAR) )                                     : 00091200   000725  UPDATE
 0016   :              IF( ABS(XT) .GT. ABS(YT) ) GO TO 30                       : 00091300   001037  UPDATE
 0018   :                Y(NPAR) = CMPLX(0., PAR)                                : 00091400   001200  UPDATE
 0019   :              GO TO 10                                                  : 00091500   001559  UPDATE
 0020   :       30       Y(NPAR) = CMPLX(PAR, 0.)                                : 00091600   001645  UPDATE
 0021   :              GO TO 10                                                  : 00091700   002004  UPDATE
 0022   :        2     RM(NPAR) = PAR                                            : 00091800   002090  UPDATE
 0023   :              GO TO 10                                                  : 00091900   002265  UPDATE
 0024   :        3     ALPHA(NPAR) = PAR                                         : 00092000   002351  UPDATE
 0025   :              GO TO 10                                                  : 00092100   002526  UPDATE
 0026   :        4     XMU(NPAR) = PAR                                           : 00092200   002612  UPDATE
 0027   :              GO TO 10                                                  : 00092300   002787  UPDATE
 0028   :        5     GM(NPAR) = PAR                                            : 00092400   002873  UPDATE
 0029   :       10     CALL YBMAT                                                : 00092850   003048  UPDATE
 0030   :              RETURN                                                    : 00092900   003153  UPDATE
 0031   :              END                                                       : 00093000   003267  UPDATE
 ERRORS AND NO WARNINGS IN 31 STATEMENTS,CODE EMITTED = 3326 BITS (416 BYTES)UPDATE
MPILE TIME IS 23.3 SECONDS FOR 40 CARDS AT 103 CARDS/MINUTE.
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-54-

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URROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,        MONDAY     10/08/79     01:43 PM
         /(E99000)    /FAULTO
  0001   :              FUNCTION F(X)                                                   : 00087920   000075  OFAULTO
         :C                                                                             : 00087940           OFAULTO
  0002   :              COMMON/SRCH/ PAR, DELPAR, YNNOM                                 : 00087960   000075  F
  0003   :              COMPLEX YNNOM(3,3)                                              : 00087980   000075  F
         :C                                                                             : 00088000           F
  0004   :              COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),    : 00088002   000075  F
  0004   :         1                 Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),    : 00088004   000075  F
  0004   :         2                 A2MNUS(50), A2PLSP(50), A2MNSP(50), I, J, IP,      : 00088006   000075  F
  0004   :         3                 N, IB, ITOT, YN(3,3)                               : 00088008   000075  F
  0005   :              INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                          : 00088010   000075  F
  0006   :              INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                          : 00088012   000075  F
  0007   :              COMPLEX YN, Y2, Y2I, Y2PI                                       : 00088014   000075  F
         :C                                                                             : 00088016           F
  0008   :              COMMON/DELP/  ITYPE, Y2IP, COSTMN, JPAR, NPAR, ICORR,           : 00088018   000075  F
  0008   :         *       IX, IW, W, WROW, WCOL, Q, XROW, XCOL                         : 00088020   000075  F
  0009   :              COMPLEX Y2IP(17,17), H(80), G(80)                               : 00088022   000075  F
  0010   :              INTEGER WROW(80), WCOL(80), XROW(80), XCOL(80)                  : 00088024   000075  F
  0011   :              INTEGER ITYPE(80), ICORR(80)                                    : 00088026   000075  F
         :C                                                                             : 00088028           F
  0012   :              DELPAR = X - PAR                                                : 00088030   000075  F
  0013   :              CALL UPDATE( X )                                                : 00088035   000427  F
  0014   :              CALL UVVEC(DELPAR, A2PLUS, A2MNUS)                              : 00088040   000568  F
  0015   :              CALL Y2PI2                                                      : 00088060   000835  F
  0016   :              CALL YHAT(A1PLUS,A1MNUS,A2PLUS,A2MNUS,Y2IP,YN,IP,IB,17)         : 00088080   000940  F
  0017   :              F = 1./COSTF(YNNOM)                                             : 00088100   001643  F
  0018   :              RETURN                                                          : 00088120   001952  F
  0019   :              END                                                             : 00088700   002066  F
O ERRORS AND NO WARNINGS IN 19 STATEMENTS,CODE EMITTED = 2125 BITS (266 BYTES)F
OMPILE TIME IS 21.8 SECONDS FOR 27 CARDS AT 74 CARDS/MINUTE.
```

```
URROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79     05:18 PM
         /(E99000)   /FAULT30
 0001  :            SUBROUTINE UVVEC( DELPAR, A2PLSP, A2MNSP )                    : 00093100     000075    OFAULT30
       :C                                                                         : 00093200               OFAULT30
 0002  :            COMMON/VECTOR/  U, NU, IU, V, NV, IV                          : 00093300     000075    UVVEC
 0003  :            COMPLEX  U(20), V(20)                                         : 00093400     000075    UVVEC
 0004  :            INTEGER IU(20), IV(20)                                        : 00093500     000075    UVVEC
       :C                                                                         : 00093600               UVVEC
 0005  :            COMMON/DELP/  ITYPE,     Y2IP, COSTMN,    JPAR,   NPAR,   ICORR,  : 00093700     000075    UVVEC
 0005  :      *           IX, IW,    W,    WROW,    WCOL,      X,    XROW,   XCOL     : 00093800     000075    UVVEC
 0006  :            COMPLEX   Y2IP(17,17), H(80), X(80)                           : 00093900     000075    UVVEC
 0007  :            INTEGER   WROW(80), WCOL(80), XROW(80), XCOL(80)              : 00094000     000075    UVVEC
 0008  :            INTEGER ITYPE(80), ICORR(80)                                  : 00094100     000075    UVVEC
       :C                                                                         : 00094200               UVVEC
 0009  :            COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),      : 00094300     000075    UVVEC
 0009  :      1                   YB(65), YROW(65), GMROW(15), GMCOL(15),         : 00094400     000075    UVVEC
 0009  :      2                   RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),   : 00094500     000075    UVVEC
 0009  :      3                   AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,  : 00094600     000075    UVVEC
 0009  :      4                   IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR         : 00094700     000075    UVVEC
 0010  :            INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,     : 00094800     000075    UVVEC
 0010  :      1           AROW, ACOL, YBROW, YBCOL                                : 00094900     000075    UVVEC
 0011  :            COMPLEX Y, YB                                                 : 00095000     000075    UVVEC
       :C                                                                         : 00095100               UVVEC
 0012  :            COMPLEX ULC(20), VLC(20), CMPLX                               : 00095200     000075    UVVEC
 0013  :            INTEGER IULC(20), IVLC(20)                                    : 00095300     000075    UVVEC
       :C                                                                         : 00095400               UVVEC
 0014  :             INTEGER A2PLSP(50), A2MNSP(50)                               : 00095500     000075    UVVEC
       :C                                                                         : 00096200               UVVEC
 0015  :            DO 10 K=1,16                                                  : 00096300     000075    UVVEC
 0016  :              ULC(K) = (0.,0.)                                            : 00096400     000330    UVVEC
 0017  :              VLC(K) = (0.,0.)                                            : 00096500     000703    UVVEC
 0018  :              U(K) = (0.,0.)                                              : 00096600     001076    UVVEC
 0019  :     10       V(K) = (0.,0.)                                              : 00096700     001449    UVVEC
 0020  :            GO TO (1,2,3,4,5), JPAR                                       : 00096800     001954    UVVEC
       :C                                                                         : 00096900               UVVEC
       :C                        ***** Y IS VARIED *****                          : 00097000               UVVEC
       :C                                                                         : 00097100               UVVEC
       :C                FORM VECTOR ULC:                                         : 00097200               UVVEC
       :C                                                                         : 00097300               UVVEC
 0021  :     1      NULC = 1                                                      : 00097400     002145    UVVEC
 0022  :            IULC(1) = NPAR                                                : 00097500     002236    UVVEC
 0023  :            ULC(1) = (1.,0.)                                              : 00097600     002350    UVVEC
       :C                                                                         : 00097700               UVVEC
       :C                FORM VECTOR VLC                                          : 00097800               UVVEC
       :C                                                                         : 00097900               UVVEC
 0024  :            NVLC = 1                                                      : 00098000     002547    UVVEC
 0025  :            IVLC(1) = NPAR                                                : 00098100     002638    UVVEC
 0026  :            VLC(1) = CMPLX( DELPAR, 0.)                                   : 00098200     002752    UVVEC
 0027  :            IF(IXMU .EQ. 0) GO TO 11                                      : 00098300     002935    UVVEC
 0029  :            DO 15 K=1,IXMU                                                : 00098400     003068    UVVEC
 0030  :              NVLC = NVLC + 1                                             : 00098500     003159    UVVEC
 0031  :              IVLC(NVLC) = XMUCOL(K)                                      : 00098600     003295    UVVEC
 0032  :              VLC(NVLC) = CMPLX( -DELPAR*XMU(K), 0.)                      : 00098700     003533    UVVEC
 0033  :     15     CONTINUE                                                      : 00098800     003996    UVVEC
 0034  :     11     IF(IRM .EQ. 0) GO TO 21                                       : 00098900     004201    UVVEC
 0036  :            DO 20 M=1,IRM                                                 : 00099000     004334    UVVEC
 0037  :              IF(RMROW(M) .NE. NPAR) GO TO 20                             : 00099100     004425    UVVEC
 0039  :              DO 14 K=1,IX                                                : 00099200     004633    UVVEC
 0040  :                IF(XROW(K) .NE. RMCOL(M)) GO TO 14                        : 00099300     004724    UVVEC
 0042  :                NVLC = NVLC + 1                                           : 00099400     004995    UVVEC
 0043  :                IVLC(NVLC) = GMCOL(K)                                     : 00099500     005131    UVVEC
 0044  :                VLC(NVLC) = CMPLX( RM(M)*XROW(K)*DELPAR, 0.)              : 00099600     005369    UVVEC
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```
      14    CONTINUE                                    005922  UVVEC
      20  CONTINUE                                      006125  UVVEC
      21  XT = REAL( Y(NPAR) )                          006328  UVVEC
          YT = AIMAG( Y(NPAR) )                         006631  UVVEC
          IF( ABS(XT) .GT. ABS(YT) ) GO TO 22           006943  UVVEC
          DO 23 K = 1,NVLC                              007106  UVVEC
      23  VLC(K) = VLC(K)*(0., 1.)                      007197  UVVEC
C                                                               UVVEC
C         NOTE: ALPHA*DELTA-Y TERMS CONSIDERED ELSEWHERE        UVVEC
C                                                               UVVEC
C         FORM VECTOR U*                                        UVVEC
C                                                               UVVEC
      22  NU = 0                                        007999  UVVEC
          NV = 0                                        008090  UVVEC
          DO 30 K=1,NULC                                008181  UVVEC
            A2 = A2PLSP(IULC(K))                        008272  UVVEC
            IF(A2 .EQ. 0) GO TO 25                      008498  UVVEC
              NU = NU + 1                               008631  UVVEC
              IU(NU) = A2                               008767  UVVEC
              U(NU) = ULC(K)                            008942  UVVEC
      25    A2 = A2MNSP(IULC(K))                        009424  UVVEC
            IF(A2 .EQ. 0) GO TO 30                      009650  UVVEC
              NU = NU + 1                               009783  UVVEC
              IU(NU) = A2                               009919  UVVEC
              U(NU) = -ULC(K)                           010094  UVVEC
      30  CONTINUE                                      010612  UVVEC
C                                                               UVVEC
C         FORM VECTOR V:                                        UVVEC
C                                                               UVVEC
          DO 40 K=1,NVLC                                010815  UVVEC
            A2 = A2PLSP(IVLC(K))                        010906  UVVEC
            IF(A2 .EQ. 0) GO TO 35                      011132  UVVEC
              NV = NV + 1                               011265  UVVEC
              IV(NV) = A2                               011401  UVVEC
              V(NV) = VLC(K)                            011576  UVVEC
      35    A2 = A2MNSP(IVLC(K))                        012058  UVVEC
            IF(A2 .EQ. 0) GO TO 40                      012284  UVVEC
              NV = NV + 1                               012417  UVVEC
              IV(NV) = A2                               012553  UVVEC
              V(NV) = -VLC(K)                           012728  UVVEC
      40  CONTINUE                                      013246  UVVEC
          RETURN                                        013449  UVVEC
C                                                               UVVEC
C              ****** RM IS VARIED: ******                      UVVEC
C                                                               UVVEC
       2  I = RMROW(NPAR)                               013563  UVVEC
          J = RMCOL(NPAR)                               013740  UVVEC
C                                                               UVVEC
C              FORM VECTOR ULC:                                 UVVEC
C                                                               UVVEC
          NULC = 1                                      013917  UVVEC
          IULC(1) = I                                   014008  UVVEC
          ULC(1) = (1.,0.)                              014122  UVVEC
C                                                               UVVEC
C              FORM VECTOR VLC:                                 UVVEC
C                                                               UVVEC
          NVLC = 0                                      014319  UVVEC
          DO 50 M=1,IX                                  014410  UVVEC
            IF(XROW(M) .NE. J) GO TO 50                 014501  UVVEC
              K = XCOL(M)                               014709  UVVEC
              NVLC = NVLC + 1                           014886  UVVEC
```

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0095 :          IVLC(NVLC) = K                          : 00105800   015022  UVVEC
0096 :          VLC(NVLC) = -Y(I)*DELPAR*X(M)           : 00105900   015197  UVVEC
0097 :    50  CONTINUE                                  : 00106000   016200  UVVEC
     :C                                                 : 00106100           UVVEC
     :C   ***  FORM VECTORS U AND V: ***                : 00106200           UVVEC
     :C                                                 : 00106300           UVVEC
0098 :        GO TO 22                                  : 00106400   016403  UVVEC
     :C                                                 : 00106500           UVVEC
     :C       ALPHA IS VARIED:                          : 00106600           UVVEC
     :C                                                 : 00106700           UVVEC
0099 :     3  I = AROW(NPAR)                            : 00106800   016489  UVVEC
0100 :        J = ACOL(NPAR)                            : 00106900   016666  UVVEC
     :C                                                 : 00107000           UVVEC
     :C              FORM VECTOR ULC:                   : 00107100           UVVEC
     :C                                                 : 00107200           UVVEC
0101 :        NULC = 1                                  : 00107300   016843  UVVEC
0102 :        IULC(1) = I                               : 00107400   016934  UVVEC
0103 :        ULC(1) = (1.,0.)                          : 00107500   017048  UVVEC
     :C                                                 : 00107600           UVVEC
     :C              FORM VECTOR VLC:                   : 00107700           UVVEC
     :C                                                 : 00107800           UVVEC
0104 :        NVLC = 0                                  : 00107900   017245  UVVEC
0105 :        DO 60 M=1,IX                              : 00108000   017336  UVVEC
0106 :          IF(XROW(M) .NE. J) GO TO 60             : 00108100   017427  UVVEC
0108 :          K = XCOL(M)                             : 00108200   017635  UVVEC
0109 :          NVLC = NVLC + 1                         : 00108300   017812  UVVEC
0110 :          IVLC(NVLC) = K                          : 00108400   017948  UVVEC
0111 :          VLC(NVLC) = -DELPAR*X(M)                : 00108500   018123  UVVEC
0112 :    60  CONTINUE                                  : 00108600   019814  UVVEC
     :C                                                 : 00108700           UVVEC
     :C       FORM VECTORS U AND V:                     : 00108800           UVVEC
     :C                                                 : 00108900           UVVEC
0113 :        GO TO 22                                  : 00109000   019017  UVVEC
     :C                                                 : 00109100           UVVEC
     :C               ***** XMU IS VARIED *****         : 00109200           UVVEC
     :C                                                 : 00109300           UVVEC
0114 :     4  I = XMUROW(NPAR)                          : 00109400   019103  UVVEC
0115 :        J = XMUCOL(NPAR)                          : 00109500   019280  UVVEC
     :C                                                 : 00109600           UVVEC
     :C              FORM VECTOR VLC:                   : 00109700           UVVEC
     :C                                                 : 00109800           UVVEC
0116 :        NVLC = 1                                  : 00109900   019457  UVVEC
0117 :        IVLC(1) = J                               : 00110000   019548  UVVEC
0118 :        VLC(1) = (1.,0.)                          : 00110100   019662  UVVEC
     :C                                                 : 00110200           UVVEC
     :C              FORM VECTOR ULC:                   : 00110300           UVVEC
     :C                                                 : 00110400           UVVEC
0119 :        NULC = 0                                  : 00110500   019859  UVVEC
0120 :        DO 70 M=1,IW                              : 00110600   019950  UVVEC
0121 :          IF(WCOL(M) .NE. I) GO TO 70             : 00110700   020041  UVVEC
0123 :          K = WROW(M)                             : 00110800   020249  UVVEC
0124 :          NULC = NULC + 1                         : 00110900   020426  UVVEC
0125 :          IULC(NULC) = K                          : 00111000   020562  UVVEC
0126 :          ULC(NULC) = -W(M)*Y(I)*DELPAR           : 00111100   020737  UVVEC
0127 :    70  CONTINUE                                  : 00111200   021786  UVVEC
     :C                                                 : 00111300           UVVEC
     :C       FORM VECTORS U AND V:                     : 00111400           UVVEC
     :C                                                 : 00111500           UVVEC
0128 :        GO TO 22                                  : 00111600   021989  UVVEC
     :C                                                 : 00111700           UVVEC
     :C               ***** GM IS VARIED *****          : 00111800           UVVEC
     :C                                                 : 00111900           UVVEC
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0129   :       5   I = GMROW(NPAR)                          : 00112000   022075   UVVEC
0130   :           J = GMCOL(NPAR)                          : 00112100   022252   UVVEC
       :C                                                   : 00112200            UVVEC
       :C                FORM VECTOR VLC:                   : 00112300            UVVEC
       :C                                                   : 00112400            UVVEC
0131   :           NVLC = 1                                 : 00112500   022429   UVVEC
0132   :           IVLC(1) = J                              : 00112600   022520   UVVEC
0133   :           VLC(1) = (1.,0.)                         : 00112700   022634   UVVEC
       :C                                                   : 00112800            UVVEC
       :C                FORM VECTOR ULC:                   : 00112900            UVVEC
       :C                                                   : 00113000            UVVEC
0134   :           NULC = 0                                 : 00113100   022831   UVVEC
0135   :           DO 80 M=1,IW                             : 00113200   022922   UVVEC
0136   :             IF(WCOL(M) .NE. I) GO TO 80            : 00113300   023013   UVVEC
0138   :               K = WROW(M)                          : 00113400   023221   UVVEC
0139   :               NULC = NULC + 1                      : 00113500   023398   UVVEC
0140   :               IULC(NULC) = K                       : 00113600   023534   UVVEC
0141   :               ULC(NULC) = W(M)*DELPAR              : 00113700   023709   UVVEC
0142   :      80   CONTINUE                                 : 00113800   024327   UVVEC
       :C                                                   : 00113900            UVVEC
       :C          FORM VECTORS U AND V:                    : 00114000            UVVEC
       :C                                                   : 00114100            UVVEC
0143   :           GO TO 22                                 : 00114200   024530   UVVEC
0144   :           END                                      : 00114300   024616   UVVEC
0 ERRORS AND NO WARNINGS IN 144 STATEMENTS.CODE EMITTED = 24730 BITS (3092 BYTES)UVVEC
OMPILE TIME IS 56.3 SECONDS FOR 207 CARDS AT 221 CARDS/MINUTE.
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-59-

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JRROUGHS B1800/B1700 FORTRAN COMPILER , MARK 8.0 RA 11/23/78 01:37 ,          TUESDAY    10/02/79    05:18 PM
   /(F99000)  /FAULT30
 0001  :           SUBROUTINE Y2PI2                                               : 00114400   000075  OFAULT30
 0002  :           COMMON/DELP/  ITYPE,    Y2IP, COSTMN,   JPAR,   NPAR,  ICORR,  : 00114500   000075  Y2PI2
 0002  :      *                   W,       WROW,   WCOL,     X,      XROW,   XCOL : 00114600   000075  Y2PI2
 0003  :           COMPLEX    Y2IP(17,17), W(80), X(80)                           : 00114700   000075  Y2PI2
 0004  :           INTEGER    WROW(80), WCOL(80), XROW(80), XCOL(80)              : 00114800   000075  Y2PI2
 0005  :           INTEGER ITYPE(80), ICORR(80)                                   : 00114900   000075  Y2PI2
       :C                                                                         : 00115000           Y2PI2
 0006  :           COMMON/VECTOR/  U, NU, IU, V, NV, IV                           : 00115100   000075  Y2PI2
 0007  :           COMPLEX  U(20), V(20)                                          : 00115200   000075  Y2PI2
 0008  :           INTEGER IU(20), IV(20)                                         : 00115300   000075  Y2PI2
       :C                                                                         : 00115400           Y2PI2
 0009  :           COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),   : 00115500   000075  Y2PI2
 0009  :      1                 Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),   : 00115600   000075  Y2PI2
 0009  :      2                 A2MNUS(50), A2PLSP(50), A2MNSP(50), I1, J1, IP,   : 00115700   000075  Y2PI2
 0009  :      3                 N, IB, ITOT, YN(3,3)                              : 00115800   000075  Y2PI2
 0010  :           INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                         : 00115900   000075  Y2PI2
 0011  :           INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                         : 00116000   000075  Y2PI2
 0012  :           COMPLEX YN, Y2, Y2I, Y2PI                                      : 00116100   000075  Y2PI2
 0013  :           COMPLEX E(20), D, ESUM                                         : 00116200   000075  Y2PI2
 0014  :           INTEGER EROW(20), ECOL(20)                                     : 00116300   000075  Y2PI2
       :C                                                                         : 00116400           Y2PI2
 0015  :           IQ = N-IP                                                      : 00116500   000075  Y2PI2
 0016  :           DO 5 I = 1,IQ                                                  : 00116600   000375  Y2PI2
 0017  :           DO 5 J = 1,IQ                                                  : 00116700   000466  Y2PI2
 0018  :      5    Y2IP(I,J) = Y2I(I,J)                                           : 00116800   000557  Y2PI2
 0019  :           D = (1.,0.)                                                    : 00116900   001431  Y2PI2
 0020  :           DO 10 M = 1,NV                                                 : 00117000   001628  Y2PI2
 0021  :           DO 10 K = 1,NU                                                 : 00117100   001719  Y2PI2
 0022  :     10    D = D + V(M)*Y2I(IV(M),IU(K))*U(K)                             : 00117200   001810  Y2PI2
 0023  :           NE = 0                                                         : 00117300   003394  Y2PI2
 0024  :           DO 30 I = 1,NU                                                 : 00117400   003485  Y2PI2
 0025  :             K = IU(I)                                                    : 00117500   003576  Y2PI2
 0026  :           DO 30 L = 1,IQ                                                 : 00117600   003753  Y2PI2
 0027  :             ESUM  =(0.,0.)                                               : 00117700   003844  Y2PI2
 0028  :             DO 20 J = 1,NV                                               : 00117800   004041  Y2PI2
 0029  :               M = IV(J)                                                  : 00117900   004132  Y2PI2
 0030  :     20      ESUM = ESUM + V(M)*Y2I(M,L)                                  : 00118000   004309  Y2PI2
 0031  :             NE = NE + 1                                                  : 00118100   005274  Y2PI2
 0032  :             E(NE) = ESUM*U(I)/D                                          : 00118200   005410  Y2PI2
 0033  :             EROW(NE) = K                                                 : 00118300   006188  Y2PI2
 0034  :     30      ECOL(NE) = L                                                 : 00118400   006363  Y2PI2
 0035  :           DO 40 K = 1,NE                                                 : 00118500   006826  Y2PI2
 0036  :               J = ECOL(K)                                                : 00118600   006917  Y2PI2
 0037  :               M = EROW(K)                                                : 00118700   007094  Y2PI2
 0038  :             DO 40 I = 1,IQ                                               : 00118800   007271  Y2PI2
 0039  :             Y2IP(I,J) = Y2IP(I,J) - Y2I(I,M)*E(K)                        : 00118900   007362  Y2PI2
 0040  :     40    CONTINUE                                                       : 00119000   008664  Y2PI2
 0041  :           RETURN                                                         : 00119100   009011  Y2PI2
 0042  :           END                                                            : 00119200   009125  Y2PI2
0 ERRORS AND NO WARNINGS IN 42 STATEMENTS,CODE EMITTED = 9184 BITS (1148 BYTES)Y2PI2
OMPILE TIME IS 37.7 SECONDS FOR 49 CARDS AT 78 CARDS/MINUTE.
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0001   :        SUBROUTINE JACCOL( DELY, IPAR)                                   : 00119300   000075  OFAULT30
       :C                                                                        : 00119400           OFAULT30
0002   :        COMMON/DELP/  ITYPE,    Y2IP, COSTMN,    JPAR,    NPAR, ICORR,   : 00119500   000075  JACCOL
0002   :     *        IX, IW,     W,  WROW,   WCOL,          X,  XROW,  XCOL     : 00119600   000075  JACCOL
0003   :        COMPLEX   Y2IP(17,17), W(80), X(80)                              : 00119700   000075  JACCOL
0004   :        INTEGER  WROW(80), WCOL(80), XROW(80), XCOL(80)                  : 00119800   000075  JACCOL
0005   :        INTEGER ITYPE(80), ICORR(80)                                     : 00119900   000075  JACCOL
       :C                                                                        : 00120000           JACCOL
0006   :        COMMON/OTHER/A1PLUS(50), A1MNUS(50), A1PLSP(50), A1MNSP(50),     : 00120100   000075  JACCOL
0006   :     1             Y2(17,17), Y2I(17,17), Y2PI(16,16), A2PLUS(50),       : 00120200   000075  JACCOL
0006   :     2             A2MNUS(50), A2PLSP(50), A2MNSP(50), I1, J1, IP,       : 00120300   000075  JACCOL
0006   :     3            NN, IB, ITDI, YN(3,3)                                  : 00120400   000075  JACCOL
0007   :        INTEGER A2PLUS, A2MNUS, A2PLSP, A2MNSP                           : 00120500   000075  JACCOL
0008   :        INTEGER A1PLUS, A1MNUS, A1PLSP, A1MNSP                           : 00120600   000075  JACCOL
0009   :        COMPLEX YN, Y2, Y2I, Y2PI                                        : 00120700   000075  JACCOL
       :C                                                                        : 00120800           JACCOL
0010   :        COMMON/ELEMNT/Y(65), GM(15), RM(15), XMU(15), ALPHA(15),         : 00120900   000075  JACCOL
0010   :     1                YB(65), YROW(65), GMROW(15), GMCOL(15),            : 00121000   000075  JACCOL
0010   :     2                RMROW(15), RMCOL(15), XMUROW(15), XMUCOL(15),      : 00121100   000075  JACCOL
0010   :     3                AROW(15), ACOL(15), YBROW(65), YBCOL(65), IGM,     : 00121200   000075  JACCOL
0010   :     4                IRM, IXMU, IA, IALPHA, IYB, OMEGA, IPAR1           : 00121300   000075  JACCOL
0011   :        INTEGER YROW, GMROW, GMCOL, RMROW, RMCOL, XMUROW, XMUCOL,        : 00121400   000075  JACCOL
0011   :     1          AROW, ACOL, YBROW, YBCOL                                 : 00121500   000075  JACCOL
0012   :        COMPLEX Y, YB                                                    : 00121600   000075  JACCOL
       :C                                                                        : 00121700           JACCOL
0013   :        COMMON/DEL/ C,D                                                  : 00121800   000075  JACCOL
0014   :        COMPLEX C(50,50), D(50,50), DELYB(80), G(50,50), F(50,50),       : 00121900   000075  JACCOL
0014   :     *          DELY(3,3), CMPLX                                         : 00122000   000075  JACCOL
0015   :        INTEGER R, DELROW(80), DELCOL(80)                                : 00122100   000075  JACCOL
       :C                                                                        : 00122200           JACCOL
       :C         INITIALIZE DELY TO ZERO:                                       : 00122300           JACCOL
       :C                                                                        : 00122400           JACCOL
0016   :        DO 10 I=1,IP                                                     : 00122500   000075  JACCOL
0017   :        DO 10 J=1,IP                                                     : 00122600   000330  JACCOL
0018   :    10  DELY(I,J) = (0.,0.)                                              : 00122700   000421  JACCOL
       :C                                                                        : 00122800           JACCOL
0019   :        IDELYB = 0                                                       : 00122900   001084  JACCOL
0020   :        I = ICORR(IPAR)                                                  : 00122950   001175  JACCOL
0021   :        IF(ITYPE(IPAR) .GE. 4) GO TO 200                                 : 00123000   001352  JACCOL
       :C                                                                        : 00123100           JACCOL
       :C         COMPUTE DELYB:                                                 : 00123200           JACCOL
       :C                                                                        : 00123300           JACCOL
       :C         PARAMETER IS PASSIVE (R, L, C):                                : 00123400           JACCOL
       :C                                                                        : 00123450           JACCOL
       :C                                                                        : 00123500           JACCOL
0023   :        DO 100 M = 1,IW                                                  : 00123600   001548  JACCOL
0024   :          R = WROW(M)                                                    : 00123700   001639  JACCOL
0025   :          J = WCOL(M)                                                    : 00123800   001816  JACCOL
0026   :          I = ICORR(IPAR)                                                : 00123900   001993  JACCOL
0027   :          IF(J .NE. I) GO TO 100                                         : 00124000   002170  JACCOL
0029   :          IDELYB = IDELYB + 1                                            : 00124100   002315  JACCOL
0030   :          DELROW(IDELYB) = R                                             : 00124200   002451  JACCOL
0031   :          DELCOL(IDELYB) = I                                             : 00124300   002626  JACCOL
0032   :          DELYB(IDELYB) = W(M)                                           : 00124400   002801  JACCOL
0033   :          DO 50 N = 1,IXMU                                               : 00124500   003283  JACCOL
0034   :            J = XMUROW(N)                                                : 00124600   003374  JACCOL
0035   :            IF(J .NE. I) GO TO 50                                        : 00124700   003551  JACCOL
0037   :            IDELYB = IDELYB + 1                                          : 00124800   003696  JACCOL
0038   :            DELROW(IDELYB) = R                                           : 00124900   003832  JACCOL
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0039  :           DELCOL(IDELYB) = XMUCOL(N)                                 : 00125000   004007  JACCOL
0040  :           DELYB(IDELYB) = -W(M)*XMU(N)                               : 00125100   004245  JACCOL
0041  :    50   CONTINUE                                                     : 00125200   005017  JACCOL
0042  :   100  CONTINUE                                                      : 00125300   005220  JACCOL
0043  :        GO TO (300, 120, 130), ITYPE(IPAR)                            : 00125400   005423  JACCOL
0044  :   120  VALUE = -1./( OMEGA*AIMAG( Y(I) ) )                           : 00125450   005641  JACCOL
0045  :        DO 125 K = 1,IDELYB                                           : 00125500   006035  JACCOL
0046  :   125  DELYB(K) = CMPLX(0., 1./(OMEGA*VALUE*VALUE) )*DELYB(K)        : 00125600   006126  JACCOL
0047  :        GO TO 300                                                     : 00125700   007140  JACCOL
0048  :   130  DO 135 K = 1,IDELYB                                           : 00125800   007226  JACCOL
0049  :   135  DELYB(K) = CMPLX(0., OMEGA)*DELYB(K)                          : 00125900   007317  JACCOL
0050  :        GO TO 300                                                     : 00126000   008215  JACCOL
      :C                                                                     : 00126100           JACCOL
      :C         PARAMETER IS AN AMPLIFIER GAIN (RM, ALPHA, MU, GM)          : 00126200           JACCOL
      :C                                                                     : 00126300           JACCOL
0051  :   200  JPAR = ITYPE(IPAR) - 10                                       : 00126400   008301  JACCOL
0052  :        IF(JPAR .EQ. 1) JPAR = 5                                      : 00126500   008500  JACCOL
0054  :        IF(JPAR .GE. 4) GO TO 250                                     : 00126600   008728  JACCOL
0056  :        IF(JPAR .EQ. 3) GO TO 201                                     : 00126700   008851  JACCOL
0058  :          M = RMROW(I)                                                : 00126800   008994  JACCOL
0059  :          N = RMCOL(I)                                                : 00126900   009171  JACCOL
0060  :        GO TO 202                                                     : 00127000   009348  JACCOL
0061  :   201    M = AROW(I)                                                 : 00127100   009434  JACCOL
0062  :          N = ACOL(I)                                                 : 00127200   009611  JACCOL
0063  :   202  DO 210 K = 1,IX                                               : 00127300   009788  JACCOL
0064  :          IF(XROW(K) .NE. N) GO TO 210                                : 00127400   009879  JACCOL
0065  :            IDELYB = IDELYB + 1                                       : 00127500   010087  JACCOL
0067  :            DELROW(IDELYB) = M                                        : 00127600   010223  JACCOL
0068  :            DELCOL(IDELYB) = XCOL(K)                                  : 00127700   010398  JACCOL
0069  :            DELYB(IDELYB) = X(K)                                      : 00127800   010636  JACCOL
0070  :          IF(JPAR .EQ. 3) DELYB(IDELYB) = -Y(M)*DELYB(IDELYB)         : 00127900   011118  JACCOL
0072  :   210  CONTINUE                                                      : 00128000   012164  JACCOL
0073  :        GO TO 300                                                     : 00128100   012367  JACCOL
0074  :   250  IF(JPAR .EQ. 5) GO TO 251                                     : 00128200   012453  JACCOL
0076  :          M = XMUROW(I)                                               : 00128300   012586  JACCOL
0077  :          N = XMUCOL(I)                                               : 00128400   012763  JACCOL
0078  :        GO TO 252                                                     : 00128500   012940  JACCOL
0079  :   251    M = GMROW(I)                                                : 00128600   013026  JACCOL
0080  :          N = GMCOL(I)                                                : 00128700   013203  JACCOL
0081  :   252  DO 260 K = 1,IW                                               : 00128800   013380  JACCOL
0082  :          IF(XCOL(K) .NE. M) GO TO 260                                : 00128900   013471  JACCOL
0084  :            IDELYB = IDELYB + 1                                       : 00129000   013679  JACCOL
0085  :            DELROW(IDELYB) = WROW(K)                                  : 00129100   013815  JACCOL
0086  :            DELCOL(IDELYB) = M                                        : 00129200   014053  JACCOL
0087  :            DELYB(IDELYB) = W(K)                                      : 00129300   014228  JACCOL
0088  :          IF(JPAR .EQ. 4) DELYB(IDELYB) = -Y(M)*DELYB(IDELYB)         : 00129400   014710  JACCOL
0090  :   260  CONTINUE                                                      : 00129500   015756  JACCOL
      :C                                                                     : 00129600           JACCOL
      :C       COMPUTE DELY/DELPAR:                                          : 00129700           JACCOL
      :C                                                                     : 00129800           JACCOL
0091  :   300  DO 310 I = 1,IB                                               : 00130000   015959  JACCOL
0092  :        DO 310 J = 1,IB                                               : 00130100   016050  JACCOL
0093  :        F(I,J) = (0.,0.)                                              : 00130200   016141  JACCOL
0094  :   310  G(I,J) = (0.,0.)                                              : 00130300   016566  JACCOL
      :C                                                                     : 00130400           JACCOL
0095  :        DO 320 J = 1,IB                                               : 00130500   017279  JACCOL
0096  :        DO 320 L = 1,IYB                                              : 00130600   017370  JACCOL
0097  :          I = YBROW(L)                                                : 00130700   017461  JACCOL
0098  :          K = YBCOL(L)                                                : 00130800   017638  JACCOL
0099  :   320  F(I,J) = F(I,J) - YB(L)*C(K,J)                                : 00130900   017815  JACCOL
0100  :        DO 330 K = 1,IB                                               : 00131000   019405  JACCOL
0101  :   330  F(K,K) = F(K,K) + (1.,0.)                                     : 00131100   019496  JACCOL
```

```
0102   :          DO 340 I = 1,IB                                        : 00131200   020377  JACCOL
0103   :          DO 340 J = 1,IB                                        : 00131300   020468  JACCOL
0104   :          DO 340 M = 1,IDELYB                                    : 00131400   020559  JACCOL
0105   :            K = DELROW(M)                                        : 00131500   020650  JACCOL
0106   :            L = DELCOL(M)                                        : 00131600   020827  JACCOL
0107   :    340   G(I,J) = G(I,J) + F(I,K)*DELYB(M)*D(L,J)               : 00131700   021004  JACCOL
       :C                                                                : 00131800           JACCOL
       :C           COMPUTE A1 * G * A1T.  A1 IS STORED SPARSELY.        : 00131900           JACCOL
       :C                                                                : 00132000           JACCOL
0108   :          DO 350 L = 1,IB                                        : 00132100   023139  JACCOL
0109   :            IPL= A1PLUS(L)                                       : 00132200   023230  JACCOL
0110   :            IM = A1MNUS(L)                                       : 00132300   023407  JACCOL
0111   :          DO 350 K = 1,IB                                        : 00132400   023584  JACCOL
0112   :            JP = A1PLUS(K)                                       : 00132500   023675  JACCOL
0113   :            JM = A1MNUS(K)                                       : 00132600   023852  JACCOL
0114   :            IF(IPL*JP .NE. 0) DELY(IPL,JP) = DELY(IPL,JP) + G(L,K)   : 00132700   024029  JACCOL
0116   :            IF(IM*JM .NE. 0) DELY(IM,JM) = DELY(IM,JM) + G(L,K)      : 00132800   025058  JACCOL
0118   :            IF(IPL*JM .NE. 0) DELY(IPL,JM) = DELY(IPL,JM) - G(L,K)   : 00132900   026087  JACCOL
0120   :            IF(IM*JP .NE. 0) DELY(IM,JP) = DELY(IM,JP) - G(L,K)      : 00133000   027116  JACCOL
0122   :    350   CONTINUE                                               : 00133100   028145  JACCOL
0123   :          RETURN                                                 : 00133230   028492  JACCOL
0124   :          END                                                    : 00133300   028606  JACCOL

NO ERRORS AND NO WARNINGS IN 124 STATEMENTS,CODE EMITTED = 28665 BITS (3584 BYTES)JACCOL
COMPILE TIME IS 59.1 SECONDS FOR 143 CARDS AT 145 CARDS/MINUTE.
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Appendix 2

Reprint of

"Efficient Fault Analysis
in
Linear Analog Circuits"

by

A. T. Johnson, Jr.

Efficient Fault Analysis in Linear Analog Circuits

ALFRED T. JOHNSON, JR., MEMBER, IEEE

Abstract—Fault analysis in analog networks is a form of network parameter identification. The problem of finding network parameters from measurements at the accessible terminals can be expressed as the solution of a system of nonlinear equations. Such a system of equations is usually solved by a multidimensional search. Every step of the search requires solving for the network responses in terms of the parameters, comparing the solution to the measured responses, and then adjusting the paremeters in such a way as to produce a response closer to the measured response. The computation time required for this analysis is often excessive in practical applications, and poses a major impediment to fault analysis in analog circuits. The representation of short and open circuits poses additional problems, depending upon how the network equations are formulated.

We have used nodal analysis, representing short circuits by a change in the network graph, and open circuits by a zero branch admittance. A formula of Householder and sparse matrix techniques are used to efficiently compute the response of an electrical circuit with either a short circuit or other large parameter change. The number of complex multiplications and divisions to find the network response at the accessible terminals when the network has a single short circuit involving at least one inaccessible terminal is reduced by about a factor of 15 compared with direct methods.

I. Introduction

It is surprising perhaps that although the problem of fault isolation in analog circuits has been studied longer than the comparable problem in digital circuits,[1] the digital problem is much better understood, and automatic test equipment (ATE) for isolating faults in digital equipment to the chip and even to the gate level is available, but ATE available for isolating faults in analog circuits is much less sophisticated [13], [14]. The explanation of this phenominon lies partly with the fact that analog signals are inherently more complex than digital signals. They occur continuously in time, rather than at discrete times, and their values (in principle) have infinite resolution, instead of being truncated to a finite number of bits.

In the linear analog circuits discussed in this report, the problem of signal variety can be partly overcome by restricting observation to the sinusoidal steady state, since the system behavior to any input can be determined from a knowledge of the sinusoidal steady state response at all frequencies. Clearly *all* frequencies cannot be considered, and the problem of how to choose a set of frequencies has only begun to receive attention [9], [10]. In addition, the response at any frequency is related to the parameter values and the network graph by nonlinear equations, to which the only method of solution possible is a time consuming search.

In this paper we show how the search for single short circuits can be accomplished efficiently enough to make the computation practical. We also demonstrate the effectiveness of this method in identifying short circuit faults in analog circuits whose resistance, capacitance, inductance, and controlled source gains differ significantly from their nominal values.

II. Formulating the Y-Parameters at the Accessible Nodes

We will consider linear lumped-parameter networks having $p+1$ accessible terminals, numbered $0-p$, as shown in Fig. 1. The network of Fig. 1 contains $n+1$ nodes, including the reference node, and b branches and is assumed to contain no internal independent sources but may contain dependent sources. We will formulate the nodal equations using the standard branch shown in Fig. 2.

The passive element in branch k, denoted by its admittance Y_{bk}, must be nonzero, but any of the sources may be zero or all may be present, if desired. In the equations that follow, v is the branch voltage vector, j is the branch current vector, e is the node voltage vector, v_s is the independent voltage source vector, and j_s is the independent current source vector, using the notation of Desoer and Kuh [1]. The dimension of each of these vectors is $b\times 1$, except e which is $n\times 1$. Considering the kth standard branch shown in Fig. 2, it is clear that the branch $v-i$ constraints are given by

$$j=j_s+(G_m+Y-Y\mu)v+(\alpha-YR_m)j-Yv_s \tag{1}$$

where $Y=\text{diag}\ (Y_k)$ is the branch passive admittance matrix, and G_m, μ, α, and R_m are the coupling matrices due to the controlled sources. The dimension of each of these matrices is $b\times b$. (The elements of matrices G_m and α are current sources, and the elements of matrices μ and R_m are voltage sources controlled, respectively, by voltages and currents.) Following the convention of Desoer and Kuh, define the $n\times 1$ node-to-branch incidence

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[1] An extensive bibliography of both digital and analog fault isolation is given by Sacks and Liberty [12].

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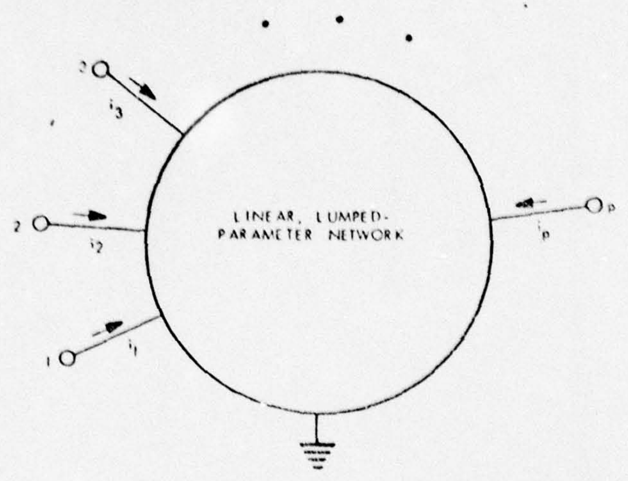

Fig. 1. Linear lumped-parameter network with $p+1$ accessible terminals.

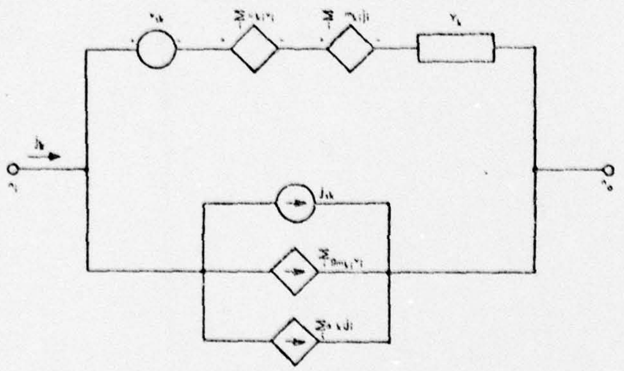

Fig. 2. Standard branch k.

matrix A,

$$\{A\}_{ij}=\begin{cases} +1, & \text{if branch } j \text{ leaves node } i\\ -1, & \text{if branch } j \text{ enters node } i\\ 0, & \text{if branch } j \text{ is not incident on node } i\end{cases}$$

Solving (1) for j and using the Kirchhoff current law $Aj=0$ and the Kirchhoff voltage law $A^Te=v$, it can be shown that the network node equations are

$$AY_bA^Te=i \tag{2}$$

where Y_b is the $b\times b$ branch admittance matrix given by

$$Y_b=(1+YR_m-\alpha)^{-1}(Y-Y\mu+G_m) \tag{3}$$

and i is the $n\times 1$ vector of independent source currents

$$i=A(1+YR_m-\alpha)^{-1}(Yv_s-j_s). \tag{4}$$

Note that Yv_s is a Norton transformation of the independent voltage sources. Because of the choice of standard branch shown in Fig. 2, the network can always be modeled with current sources only..

In both (3) and (4) it is necessary to invert the matrix $1+YR_m-\alpha$ in order to formulate the node equations. Johnson and Pennington [2] have shown that if the network can be modeled so that the branch current j_k of a branch which contains a current controlled source does not control any source, then the inverse of $1+YR_m-\alpha$ is $1-YR_m+\alpha$. Since most networks can be modeled this way, forming the node equations does not present any difficulty.

The accessible nodes have been numbered 0–p, and the inaccessible nodes have been numbered $p+1$ through n. Partition the rows of the node-to-branch incidence matrix A into accessible and inaccessible nodes

$$A=\left[\begin{array}{c}A_1\\ \hline A_2\end{array}\right] \tag{5}$$

where A_1 is the $p\times n$ incidence matrix of accessible nodes, and A_2 is the $q\times n$ incidence matrix of inaccessible nodes, and $q=n-p$ is the number of inaccessible nodes. Using (5) in (2) and partitioning i and e as follows:

$$i=\left[\begin{array}{c}i_1\\ \hline i_2\end{array}\right] \qquad e=\left[\begin{array}{c}e_1\\ \hline e_2\end{array}\right]$$

where i_1 and e_1 are $p\times 1$ vectors, and i_2 and e_2 are $q\times 1$ vectors, we obtain

$$\left[\begin{array}{c|c}A_1Y_bA_1^T & A_1Y_bA_2^T\\ \hline A_2Y_bA_1^T & A_2Y_bA_2^T\end{array}\right]\left[\begin{array}{c}e_1\\ \hline e_2\end{array}\right]=\left[\begin{array}{c}i_1\\ \hline i_2\end{array}\right]. \tag{6}$$

Now eliminate e_2 from (6) and make use of the fact that $i_2=0$, since no independent sources are connected to inaccessible nodes, to obtain

$$i_1=A_1Y_b\left[1-A_2^T(A_2Y_bA_2^T)^{-1}A_2Y_b\right]A_1^Te_1 \tag{7}$$

so that

$$y_i\triangleq A_1Y_b\left[1-A_2^T(A_2Y_bA_2^T)^{-1}A_2Y_b\right]A_1^T \tag{8}$$

is the $p\times p$ matrix of measurable y-parameters defined at the ports having a common "ground" node (node 0 in Fig. 1). The subscript i indicates that the y-parameters are evaluated at frequency ω_i.

III. Fault Analysis

Let $\hat{y}_i$ be the matrix of measured y-parameters, where the measurement is made at frequency ω_i. A fault represented by a shift of the network element values from their nominal values could be identified by solving the nonlinear equation

$$\hat{y}_i=y_i(Y_b) \tag{9}$$

for the unknown parameter values. If a single frequency is used, as in (9), the number of network parameters usually exceeds the number of independent y-parameters, so that (9) has no unique solution. One approach, suggested by Ransom and Saeks [3], is to find the solution to (9) that minimizes a norm $\|Y_{b0}-Y_b\|$, where Y_{b0} is evaluated from the nominal network element values. Ransom and Saeks give an approximate solution to this problem using linear methods. The difficulty with this approach is that the solution assumes, roughly speaking, that the most likely state of the element values in the network is the one which causes the smallest drift from the nominal values consistent with the measurements. This assumption excludes the possibility of catastrophic faults (open and short circuits).

Another approach is to augment (9) by measurements at a sufficient number of frequencies that a unique solution exists. The relationship between fault resolution and the number of frequencies and measurements is discussed by Sen and Saeks [9], [10]. Although their algorithm requires choosing a number of frequencies, it is not clear how this is done. Thus we might define:

$$\hat{y}=(\hat{y}_1\,\hat{y}_2\cdots\hat{y}_k)^T$$

$$y=(y_1\,y_2\cdots y_k)^T \tag{10}$$

and solve the equation

$$\hat{y}=y(Y_b). \tag{11}$$

Nonlinear equations, such as (11), are often solved by defining a norm $c=\|\hat{y}-y\|$, and then making a multidimensional search for min c. The amount of computation required by such a search could be prohibitive. Chen and Saeks [4] show that if only one parameter is changed at a time, the measurable system function can be evaluated without the need to invert a matrix, which is apparently necessary in (8). Their method, is based on a formula given by Householder [5].

In this paper we show how the Householder formula can be used to efficiently analyze networks containing short circuits by considering the change in the network graph. The measurable effects of open circuits and finite parameter changes can be analyzed by considering changes in Y_b, and although this is essentially equivalent to the method described by Chen and Saeks, it is given here in the context of nodal analysis so that all parameter changes can be analyzed with one formulation of the network equations.

A. Short Circuits

If two nodes of a network become shorted, the new node-to-branch incidence matrix A' is related to the original incidence matrix in a very simple way. If nodes i and j are shorted, then A' is formed by replacing row i by the sums of rows i and j, and then deleting row j. If node i is the ground node, then A' is formed by deleting row j. These row transformations can be obtained by premultiplying A by a matrix R, each element of which is either 0 or 1. Fig. 3 illustrates the row transformation matrix describing two different short circuits (one involving the ground node and the other not involving the ground node) in a network with 5 nodes and 7 branches. The incidence matrices are partitioned by the dashed lines under the assumption that nodes 3 and 4 in the original network are inaccessible. Thus the top half of the A matrix is A_1 and the bottom half is A_2. A similar partition is shown for $A'=(A_1' \mid A_2')^T$, but in the transformed network some of the formerly inaccessible nodes may have been made accessible (by virtue of their having been shorted to an accessible node).

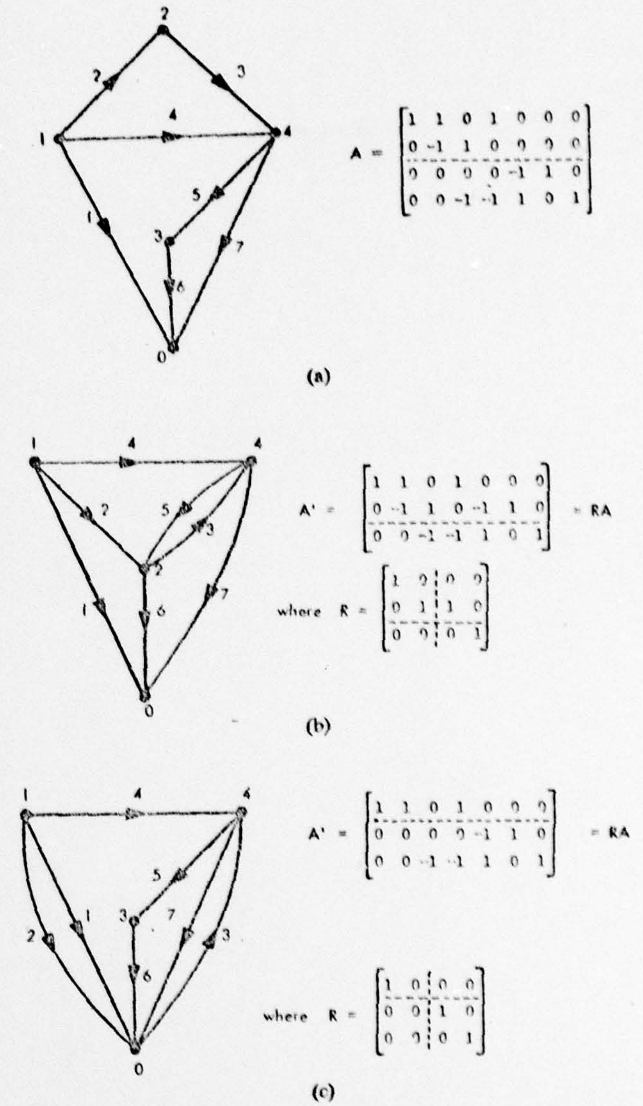

Fig. 3. Effect of short circuits on node-to-branch incidence matrix, A. (a) Original network graph. (b) Short circuit between nodes 2 and 3. (c) Short circuit between nodes 2 and ground or 0.

The R matrix is the "after-to-before" node transformation matrix, which has a column for every original node in the network and a row for every node after the fault. (Since node 0 does not enter into A, it does not appear in R either.) Thus R may be partitioned by columns and rows into the accessible and inaccessible nodes before (columns) and after (rows) the fault, as shown in Fig. 3.

Denote the partitioning of R by

$$R \triangleq \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \tag{12}$$

where the lower, left partition in (12) is $\mathbf{0}$ since the faulted network cannot have any inaccessible nodes which were accessible in the unfaulted network. In other words, if a short circuit occurs between an accessible and an inaccessible node, the combined node becomes accessible. Thus

$$A' = RA = \begin{bmatrix} R_{11}A_1 + R_{12}A_2 \\ \hdashline R_{22}A_2 \end{bmatrix}$$

and, therefore,

$$\begin{aligned} A_1' &= R_{11}A_1 + R_{12}A_2 \\ A_2' &= R_{22}A_2. \end{aligned} \tag{13}$$

B. Forming the y-Parameters of the Shorted Network

The y-parameters of networks containing a short circuit can be computed from (8) after replacing A_1 by A_1' and A_2 by A_2'. Define $Y_2 \triangleq A_2 Y_b A_2^T$, which is the $q \times q$ nodal admittance matrix of inaccessible nodes. In the analysis that follows, we assume that Y_2 is known. (In fact Y_2^{-1} must be computed only once for any circuit using the nominal element values, so the cost of its computation is a negligible part of the total cost of analysis.)

If the short circuit occurs between two accessible nodes, then $A_2' = A_2$, and the new y-parameters are found from (8) with no matrix inversion. In fact, if a short circuit exists between two accessible nodes, it can be easily observed by making resistance measurements between all pairs of accessible nodes, so the method described here is not needed.

If the short circuit involves any inaccessible node, then $A_2' \neq A_2$ and the inverse of $Y_2' = A_2' Y_b A_2'^T$ must be computed. The main contribution of this paper is to show that the inverse of Y_2' may be found in terms of Y_2 with about one fifteenth as many multiplications as are required to find the inverse of a general $q \times q$ matrix. This saving, described in detail in the next section, is due to the fact that Y_2' is closely related to Y_2, whose inverse is kown. An additional saving of computation and memory is realized by the use of sparse matrix techniques, which are discussed in Section IV.

C. Finding the Inverse of Y_2' Efficiently

From the definitions of A_2', R_{22}, Y_2, and Y_2' it is easy to show that

$$Y_2' = R_{22} Y_2 R_{22}^T. \tag{14}$$

Let us add a row to R_{22} as follows

$$S \triangleq \begin{bmatrix} R_{22} \\ \hdashline u^T \end{bmatrix} \tag{15}$$

where u^T is a row unit vector containing all zeros except for a 1 in the right-most column representing a shorted node. (At most two columns of R_{22} are involved in the short circuit. If only one is involved, then that column of u^T is unity. If two columns of R_{22} are involved, then only the right-most one is unity. This convention could be reversed by changing the definition of R.) We now decompose S into U and P,

$$S = U + P \tag{16}$$

where P is formed as follows. If the short circuit is between an accessible and an inaccessible node, then $P = 0$. If the short circuit is between two inaccessible nodes, then $\{P\}_{i-p,j-p}$ is unity, and the remaining elements are zero. (In this case, the unit element in P is element $\{R\}_{ij}$, where nodes i and j are shorted, and $i<j$ due to our numbering convention.) With these definitions, it is easy to see that U is orthogonal, since its columns are independent unit vectors and thus form an orthonormal se' [7]. Consequently $U^{-1} = U^T$, and if the short circuit is between an accessible and an inaccessible node, then $S^{-1} = S^T$. If the short circuit is between two inaccessible nodes, then S^{-1} is only slightly more difficult to compute, as we will show. Now define $C \triangleq S Y_2 S^T$, and, therefore,

$$C = \begin{bmatrix} R_{22} \\ \hdashline u^T \end{bmatrix} Y_2 \begin{bmatrix} R_{22}^T & \vdots & u \end{bmatrix} = \begin{bmatrix} Y_2' & \vdots & R_{22}Y_2u \\ \hdashline u^T Y_2 R_{22}^T & \vdots & u^T Y_2 u \end{bmatrix}$$

and

$$C^{-1} = (S^T)^{-1} Y_2^{-1} S^{-1} \triangleq \begin{bmatrix} K & \vdots & L \\ \hdashline M & \vdots & N \end{bmatrix}. \tag{17}$$

Faddeeva [8] shows that $K = Y_2'^{-1}[1-(R_{22}Y_2u)M]$, and, therefore,

$$Y_2'^{-1} = K[1-(R_{22}Y_2u)M]^{-1}. \tag{18}$$

The inverse on the right side of (18) can be computed efficiently using a formula given by Householder [5], [6], which states that if $A = B - vw^T$, where v and w are column vectors, then

$$A^{-1} = B^{-1} + \frac{B^{-1}vw^T B^{-1}}{1 - w^T B^{-1} v} \tag{19}$$

where the division on the right side of (19) is by a scalar. Thus the inverse of Y_2' is

$$Y_2'^{-1} = K\left[1 + \frac{(R_{22}Y_2u)M}{1 - M(R_{22}Y_2u)}\right]. \tag{20}$$

In order to use (20), K and M must be found from (17). We have mentioned that if the short circuit is between an accessible node and an inaccessible node, then $S^{-1} = S^T$. In fact, if the short circuit is between an accessible node and node n, then $S = I$, and K and M are simply partitions of (17). If the short circuit is between two inaccessible nodes, we have shown that $S = U + P$, where $U^{-1} = U^T$, and the rank of P is unity. Thus P can be written in the form vw^T, and (19) can be used to find S^{-1}. Proceeding in this manner it can be shown that $S^{-1} = U^T + Q$, where

$$Q_{rs} = \begin{cases} -1, & \text{if } r = i-p,\ s = n-p \\ 0, & \text{otherwise} \end{cases}.$$

And so, even in the most complex case, K and M can be evaluated without a matrix inversion. Some time can be saved in computing the triple matrix product in (17) by noting that L and N (each column vectors) are not needed, so the last column of C^{-1} need not be evaluated. Using straightforward matrix multiplication to find K and

M from (17) and then $Y_2'^{-1}$ from (20) requires $q^2(q^4-2q^3+3q^2-q+1)$ complex multiplications, where q is the number of accessible nodes in the network.

This method of computing $Y_2'^{-1}$ is unnecessarily inefficient since many of the matrices in (17) and (20) are sparse, and others (S and R_{22}), in addition, are special since their nonzero elements are either $+1$ or -1. In Secton IV-A we show how to reduce the number of complex multiplications required to evaluate $Y_2'^{-1}$ to $\beta q^2(q-1)$, where β is typically about 0.3 and is always a positive number less than unity.

Once $Y_2'^{-1}$ is known, the y-parameters at the accessible nodes are found from (8). Evaluating the matrix products in (8) in a straightforward manner requires $2b^3+(2p+q-1)b^2+(q-1)^2b$ multiplications, where b is the number of branches in the network, p is the number of accessible nodes not counting the reference node, and q is the number of inaccessible nodes. Using the sparse matrix techniques described in Section IV-A, the number of multiplications needed to evaluate y_i from $Y_2'^{-1}$ can be reduced to $2(1+\gamma)b^2$, where γ is typically about 0.1. For example, a network with 20 branches, 3 accessible nodes (plus the accessible reference node), and 8 inaccessible nodes requires 14180 multiplications to evaluate y_i directly from (8). By using sparse matrix techniques, this is reduced to 880 multiplications, a saving of a factor of 16 in this case. The straightforward evaluation in general requires nearly b times as many multiplications as the more efficient approach using sparse matrix techniques, so the saving increases with network complexity.

D. Open Circuits and Other Large Parameter Changes

For completeness, we include a discussion of open circuits and other large parameter changes, although this has been discussed elsewhere [4]. The main purpose in this section is to show how the analysis can be carried out in the context of nodal analysis.

Parameter changes are accounted for in (8) by a change in the $b\times b$ branch admittance matrix Y_b. In order to evaluate the new y-parameters from (8), the inverse of the inaccessible nodal admittance matrix $Y_2=A_2Y_bA_2^T$ must be found. If a single parameter in the network is changed, then the new branch admittance matrix is $Y_b' \triangleq Y_b+\Delta$, where Δ has only one nonzero element representing the change in one parameter. Suppose a change occurs in element ij. Then Δ can be written duv^T, where d is a scalar equal to the parameter change, u is a unit vector in direction i, and v is a unit vector in direction j. Now define $U\triangleq A_2u$ and $V\triangleq A_2v$, so that $Y_2'=Y_2+dUV^T$. Householder shows that the inverse of Y_2' can be found from

$$Y_2'^{-1}=Y_2^{-1}\left[1-U\left(d^{-1}+V^TY_2^{-1}U\right)^{-1}V^TY_2^{-1}\right]. \quad (21)$$

The inverse of Y_2 is known, and $d^{-1}+V^TY_2^{-1}U$ is a scalar, so (21) can be evaluated without a matrix inversion.

IV. Sparse Matrix Techniques

Sparse matrices arise in both the evaluation of the inverse if Y_2' and in the evaluation of the accessible y-parameter matrix y_i. These are discussed separately in Sections IV-A and IV-B.

A. Evaluating the Inverse of Y_2'

We have shown that the inverse of Y_2' may be found from (20), which is repeated below for convenience:

$$Y_2'^{-1}=K\left[1+(R_{22}Y_2u)M/(1-M(R_{22}Y_2u))\right]$$

where K and M are defined by

$$\begin{bmatrix} K & L \\ M & N \end{bmatrix}=(S^T)^{-1}Y_2^{-1}S^{-1}. \quad (22)$$

The dimensions of K and M are, respectively, $(q-1)\times(q-1)$ and $1\times(q-1)$.

In (22) Y_2^{-1} is known, and S is defined by

$$S=\begin{bmatrix} R_{22} \\ u^T \end{bmatrix}. \quad (23)$$

R_{22} is the matrix which transforms A_2, the (inaccessible node)-to-branch incidence matrix before the fault, into A_2', the (inaccessible node)-to-branch incidence matrix after the fault, so that

$$A_2'=R_{22}A_2. \quad (24)$$

In the discussion that follows, the reader may find it helpful to refer to the illustration of Fig. 3. Suppose nodes i and j are shorted, where $j>i$. Then rows 1 through $i-1$ of R_{22} contain zeros except that the diagonal element is unity. This indicates that nodes 1 through $i-1$ in the faulted network are identical to nodes 1 through $i-1$ in the unfaulted network. Row i of R_{22} contains all zeros except for unity elements in columns i and j, indicating that node i in the faulted network is formed by combining nodes i and j in the unfaulted network. Finally, the remaining rows of R_{22} contain all zeros, except that the first row to the right of the diagonal is unity. This indicates that the remaining nodes of the faulted network are identical to the remaining nodes of the unfaulted network. The faulted network contains one less node than the unfaulted network, so the dimension of R_{22} is $(q-1)\times q$, where, as usual, q is the number of inaccessible nodes.

We have chosen to number the nodes consecutively, beginning with the accessible nodes. We have also avoided analyzing networks with a short circuit between two accessible nodes, since such a fault may be found from driving point impedance measurements between all pairs of accessible nodes. Thus node j is always inaccessible, but node i may be either accessible or inaccessible. In the description above we have assumed that node i is inaccessible. If it is accessible, then R_{22} is simpler than described above, since all rows contain a single unit element. (The unit element is on the diagonal in rows 1 through $j-1$ and in the first column to the right of the diagonal in all other rows. This indicates that all inaccessible nodes in the faulted network correspond to an inacces-

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0 \end{bmatrix}$$

(b)

Fig. 4. S matrices for a network with 8 accessible nodes. (a) Short circuit between nodes 4 and 6. (b) Short circuit between nodes 6 and any accessible node.

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&-1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

(b)

Fig. 5. (a) Inverse of the matrix of Fig. 4(a). (b) Inverse of the matrix of Fig. 4(b).

sible node in the unfaulted network, but one inaccessible node in the unfaulted network is now accessible, and so it has no row in R_{22}.)

In (23) u^T is a row vector containing all zeros except that column j is unity.

To illustrate the formation of the S matrix, consider a network with 8 inaccessible nodes. (For convenience in this discussion they are numbered 1–8, instead of $p+1$ through $p+8$.) Fig. 4(a) shows the S matrix for a short circuit between nodes 4 and 6, and Fig. 4(b) shows the S matrix for a short circuit between node 6 and any accessible node. The formation of the inverse of the S matrix is given by $S^{-1} = U^T + Q$, where

$$[Q]_{rs} = \begin{cases} -1, & \text{if } r = i-p,\ s = n-p \\ 0, & \text{otherwise} \end{cases}$$

as explained in Section III-C, where $S = U + P$, and all elements of P are zero except if the shorted nodes i and j are both inaccessible, in which case $[P]_{ij}$ is unity. The unity element of P is circled in Fig. 4(a). In this illustration we have taken $p=0$ for convenience, where p is the number of accessible nodes. Also note that $n-p=q$. The inverse of the S matrix can be formed by inspection, and in particular, Fig. 5 shows the inverses of the matrices of Fig. 4.

We now turn to the problem of evaluating K and M. The right side of (22) may be written

$$(U+Q^T)Y_2^{-1}(U^T+Q) = UY_2^{-1}U^T + UY_2^{-1}Q + Q^TY_2^{-1}U^T + Q^TY_2^{-1}Q. \quad (25)$$

Recall that the right-most column (column q) of (25) is not needed. The second and fourth terms on the right side of (25) are zero except for the right-most column, so they need not be evaluated.

To evaluate $UY_2^{-1}U^T$, define the row vector $V^T \triangleq [v_k]$, where $v_k =$ (unit column of row k in U), $k = 1, 2, \cdots, q-1$. Thus

$$[K]_{mn} = [UY_2^{-1}U^T]_{mn} = [Y_2^{-1}]_{v_m, v_n}, \quad \begin{matrix} m = 1, 2, \cdots, q-1 \\ n = 1, 2, \cdots, q-1 \end{matrix}. \quad (26)$$

Finally M is the $1 \times (q-1)$ row vector whose elements are the first $q-1$ columns of the last row (row q) of the sum of terms one and three of (25). Thus

$$[M]_n = [UY_2^{-1}U^T]_{q,n} + [Q^TY_2^{-1}U^T]_{q,n} = [Y_2^{-1}]_{j,v_n} - [Y_2^{-1}]_{i,v_n}, \quad n = 1, 2, \cdots, q-1. \quad (27)$$

K and M are evaluated from (26) and (27) with no multiplications, and the evaluation of K, the larger of the two, requires no additions.

Now that K and M are known, $Y_2'^{-1}$ can be found from (21). In evaluating (21) it is useful to first find the $(q-1) \times 1$ column vector $x \triangleq R_{22}Y_2u$. Recall that u is a vector with all zero elements except that row j is unity. Thus x is the jth column of $R_{22}Y_2$. R_{22} is the partition of S formed from the first $q-1$ rows (23). We may make use of the row vector V^T to form x as follows:

$$[x]_n = [R_{22} Y_2 u]_n = \begin{cases} [Y_2]_{v_n,j}, & \text{if node } i \text{ is accessible} \\ \left\{ \begin{array}{ll} [Y_2]_{v_n,j}, & v_n \neq i \\ [Y_2]_{v_n,j} + [Y_2]_{j,j}, & v_n = i \end{array} \right\}, & \text{if node } i \text{ is inaccessible} \end{cases}$$
$$n = 1, 2, \cdots, q-1. \quad (28)$$

Using the definition of x in (28), (21) may be written

$$Y_2'^{-1} = K + K[xM/(1-Mx)] \quad (29)$$

which requires evaluating three matrix products, two of which directly involve x. From (28) it is clear that x is formed by reordering the elements of column j of Y_2. Since only a few inaccessible nodes are normally coupled to node j, x is usually sparse. Define β equal to the fraction of nonzero elements of column j of Y_2. The number of multiplications required to compute Mx is $\beta q < q$. β is usually significantly less than one, and so it is worth designing the algorithm to avoid the multiplications by zero. Similarly, βq rows of xM are zero, and it is worth forming only the nonzero rows. Thus (29) can be evaluated with $\beta q^2(q-1)$ multiplications and divisions. Note that the multiplications in (29) are the only multiplications required to evaluate $Y_2'^{-1}$, since K and M have been formed in (26) and (27) without multiplications.

B. Evaluating the y-Parameters

In this section we discuss the evaluation of the y-parameter matrix y_i given by (8). In the faulted network, the node-to-branch incidence matrices are A_1' and A_2', so that (8) becomes

$$y_i = A_1' Y_b [1 - A_2'^T Y_2'^{-1} A_2' Y_b] A_1'^T. \quad (30)$$

Two forms appear in (30) which must be discussed separately: $A^T Y A$ and $A Y A^T$.

The most convenient way to store the node-to-branch incidence matrices, which we will refer to generally by A in the discussion which follows, is by two column vectors A^+ and A^-, defined as follows. If the kth column of A has a $+1$ element, then $[A^+]_k \triangleq$ (row number of the $+1$ element of the kth column of A), otherwise $[A^+]_k \triangleq 0$. Similarly, if the kth column of A has a -1 element, then $[A^-]_k \triangleq$ (row number of the -1 element of the kth column of A), otherwise $[A^-]_k = 0$. The formation of A^+ and A^- corresponding to a given node-to-branch incidence matrix A is illustrated in Fig. 6. Note that in practice it is necessary to store only the vectors A^+ and A^- and not the matrix A.

Now consider the evaluation of $X = A^T Y A$. Element $[X]_{m,n}$ is given by

$$\begin{aligned} [X]_{m,n} &= \sum_k [A^T]_{m,k} \sum_l [Y]_{k,l} \cdot [A]_{l,n} \\ &= \sum_{k,l} [A]_{k,m} \cdot [A]_{l,n} \cdot [Y]_{k,l} \\ &= [Y]_{[A^+]_m,[A^+]_n} + [Y]_{[A^-]_m,[A^-]_n} \\ &\quad - [Y]_{[A^+]_m,[A^-]_n} - [Y]_{[A^-]_m,[A^+]_n}. \end{aligned} \quad (31)$$

$$A = \begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \\ 3 \\ 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} \qquad A^- = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \\ 5 \\ 4 \\ 4 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

Fig. 6. Node-to-branch incidence matrix A, and the corresponding compact equivalent vectors A^+ and A^-.

If any of the subscripts of $[Y]$ in the last expression is zero, the term is zero.

The evaluation of $X' = AYA^T$ is somewhat different because the summation is along the rows of A instead of down the columns. In particular, element $[X']_{m,n}$ is given by

$$\begin{aligned} [X']_{m,n} &= \sum_k [A]_{m,k} \sum_l [Y]_{k,l} \cdot [A^T]_{l,n} \\ &= \sum_{k,l} [A]_{m,k} \cdot [A]_{n,l} \cdot [Y]_{k,l}. \end{aligned} \quad (32)$$

The right side of (32) cannot be evaluated by choosing m and n as in (31). Instead k and l are chosen and, for each choice, $[Y]_{k,l}$ is added to elements $[X']_{[A^+]_k,[A^+]_l}$ and $[X']_{[A^-]_k,[A^-]_l}$ and subtracted from elements $[X']_{[A^+]_k,[A^-]_l}$ and $[X']_{[A^-]_k,[A^+]_l}$. If any of the subscripts of $[X']$ is zero, that term is zero.

Products of the forms $X' = AYA^T$ and $X = A^T YA$ both appear in (30), and we have shown how to evaluate them without multiplication. An additional product of the form $X' \cdot Y_b$ must be evaluated, where Y_b is given by (3). Y_b is typically very sparse, so in the software we have chosen to store it as three vectors: one containing the nonzero elements of Y_b, and the other two containing, respectively, the row and column of that element. The saving in storage can be appreciated by recognizing that Y_b is a $b \times b$ matrix, where b is the number of branches in the network. The elements of Y_b which are not zero are the b diagonal elements plus one additional element for each controlled source. If the number of controlled sources is $b/10$, then

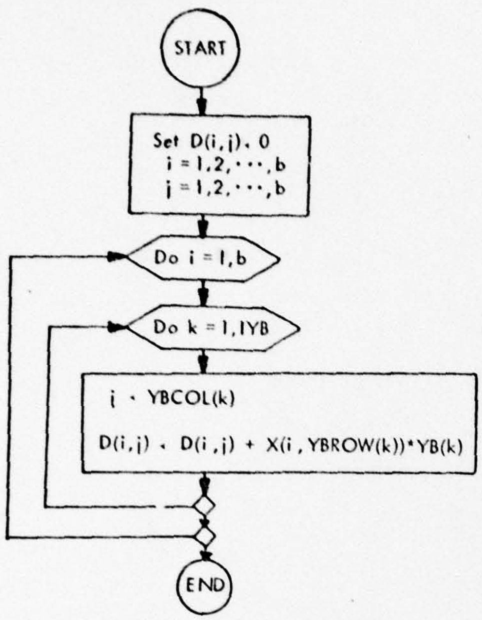

Fig. 7. Efficient formation of the matrix product $X' \cdot Y_b$.

the sparse storage of Y_b requires storing $1.1b$ complex numbers and $2.2b$ integers, compared with the b^2 complex numbers required for conventional storage. If the software is written to accommodate networks with 100 branches, then conventional storage requires setting aside room for 10^4 complex numbers, compared to about 120 complex numbers and 240 integers required by compact storage.

Considerable computation time is also saved by storing Y_b sparsely. The matrix product $X' \cdot Y_b$ in (30) would require b^3 complex multiplications if performed directly, since the dimensions of both X' and Y_b are $b \times b$. The algorithm of Fig. 7 forms the product in $(1+\gamma)b^2$ complex multiplications, where γ is the fraction of the b branches containing a controlled source. Typically γ is about 0.1. In Fig. 7, $YB(\cdot)$ is a vector containing the IYB nonzero elements of Y_b, and $YBROW(\cdot)$ and $YBCOL(\cdot)$ are integer vectors containing the row and column numbers of each element of $YB(\cdot)$. The product $X' \cdot Y_b$ is called $D(\cdot,\cdot)$. $D(\cdot,\cdot)$ is not sparse, so it is stored conventionally.

Examination of (30) reveals another matrix product of the form $Y_b \cdot Z$, where in this case $Z = 1 + A_2^T Y_2'^{-1} A_2' Y_b$. This matrix product is formed in a similar manner to $X' Y_b$ in $(1+\gamma)b^2$ complex multiplications. Thus altogether $2(1+\gamma)b^2$ complex multiplications are required to formulate y_i. A direct evaluation of (30) requires $2b^3 + (q-1+p)b^2 + ((q-1)^2 + p^2)b$ multiplications. Taking $p=2$, $b=20$, $q=11$ we find that y_i requires 22880 multiplications by direct methods, but only 880 multiplications by the methods described in this section with $\gamma = 0.1$, a saving of a factor of 26 in this example.

Before closing this section it should be noted that the y-parameters of the unfaulted network are evaluated before evaluating the y-parameters of any faulted networks. This is done by applying (30) with A_1' replaced by A_1, A_2' replaced by A_2, and $Y_2'^{-1}$ replaced by Y_2^{-1}. The inverse of Y_2 must be computed by conventional methods, but the methods of this section may be used for the remaining computation.

C. Summary of Efficient Computation

The y-parameters at the accessible ports of the faulted network are found from (30), where in the context of this report the faulted network is formed from the unfaulted network by shorting a single pair of nodes.

In Section IV-A we showed how to form the inverse of the $(q-1)\times(q-1)$ matrix Y_2' with only $\beta q^2(q-1)$ complex multiplications, where β is the fraction of nonzero elements in Y_2. (Recall that $Y_2 = A_2 Y_b A_2^T$ is the inaccessible nodal admittance matrix.) Inversion of a $(q-1)\times(q-1)$ matrix requires about $5(q-1)^3$ complex multiplications using Wilf's rank annihilation method[2] [11], which is an efficient application of the Householder formula (19) [5]–[7].

In Section IV-B we showed that (30) can be evaluated in $2(1+\gamma)b^2$ complex multiplications. This result applies, whether or not the efficient evaluation of the inverse of Y_2' of Section IV-A is used. Thus the total number of complex multiplications required to evaluate the y-parameters at the accessible ports of the network with one pair of shorted terminals is $2(1+\gamma)b^2 + q^2(q-1)$. If the inverse of Y_2' is found by conventional methods but the sparse matrix techniques of Section IV-B are used to form y_i, then $2(1+\gamma)b^2 + 5(q-1)^3$ complex multiplications are required.

The y-parameters resulting from all possible single short circuits involving at least one inaccessible node must be evaluated. If the network contains p accessible terminals and q inaccessible terminals, then it can be shown that $N = q(p+q) - q(q-1)/2$ distinct pairs of shorted terminals can be identified. The total number of complex multiplications is, therefore, $N[2(1+\gamma)b^2 + \beta q^2(q-1)]$ using the efficient method in inverting Y_2' and $N[2(1+\gamma)b^2 + 5(q-1)^3]$ using a conventional inversion method. To show the benefit of the efficient matrix inversion, let us assume that the number of accessible ports p is 2, the number of branches b is twice the number of nodes, $p+q+1$, the fraction γ of branches having controlled sources is 0.1, and the fraction β of nonzero elements of Y_2' is 0.3. Using these assumptions, the total number of complex multiplications necessary to analyze all the inaccessible short circuits using both the efficient and the conventional methods of inverting Y_2' are shown in Fig. 8. The advantage of the efficient method is especially noticeable for the larger networks.

V. Example

We have described an efficient way of computing the y-parameters at the accessible terminals of a network with

[2]An error appears in our edition of Ralston and Wilf [11]. In the flow chart on p. 75, step 11 should be $1 \to i$, not $2 \to i$.

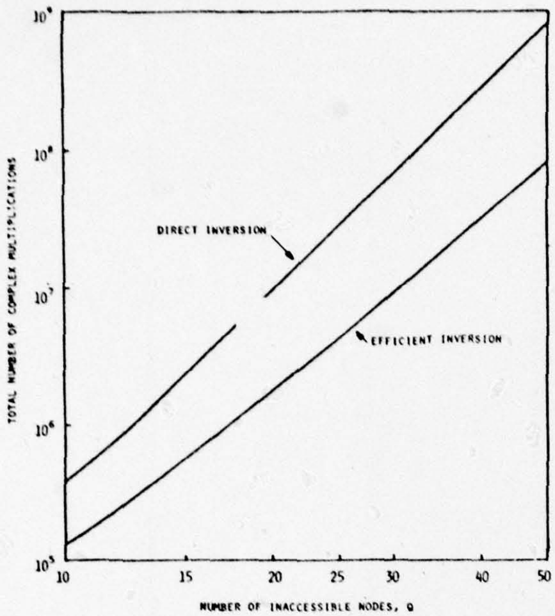

Fig. 8. Number of complex multiplications required to analyze all inaccessible short circuits in a network with q inaccessible nodes.

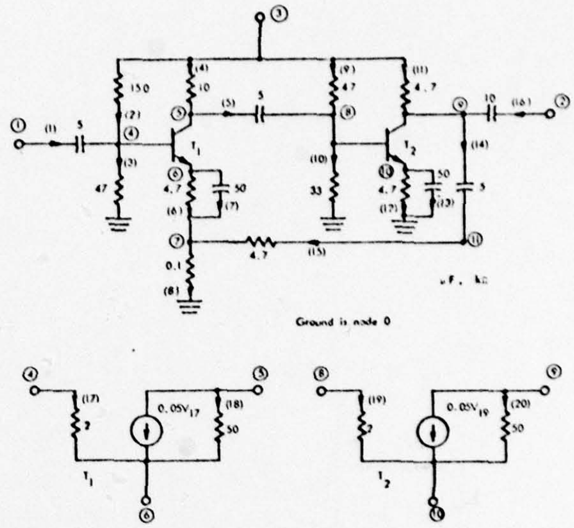

Fig. 9. Feedback amplifier.

TABLE I
SUMMARY OF TEST RESULTS I

FREQUENCY, Hz	ACTUAL SHORTED NODES	SHORTED NODES OF MINIMUM COST FUNCTION	MINIMUM COST FUNCTION VALUE	COST FUNCTION VALUE OF CORRECT FAULT
1000	0-8	0-8	1.087	1.087
	0-5	0-5	1.115	1.115
	0-10	0-10	1.290	1.290
	6-7	5-8	1.271	1.34
	3-5	3-8	1.122	1.20
	3-9	3-9	2.828	2.828
	7-8	7-8	1.632	1.632

an internal short circuit. The question remains how useful this is in locating short-circuit faults, since the network parameters (resistance, capacitance, inductance, and controlled source gains) of the faulted network will be different from their nominal values. One possible way to locate the fault is to simulate all single short circuits (involving at least one inaccessible terminal) using the nominal parameters and the algorithm described in this paper, compare each of the simulated y-parameters to the measured y-parameters, and choose the closest set to locate the short circuit. Next a search over the parameter values could be conducted, as described by Chen and Saeks [4] or in Section III-D.

To date the short circuit algorithm has been tested using the network of Fig. 9. Measured data for the nine complex y-parameters at the three accessible ports (nodes 1-0, 2-0, and 3-0) were obtained by simulating single short circuits and setting the network parameters between 10 and 20 percent above or below nominal. It was expected that this might completely mask the effect of some short circuits. The cost function used to compare the "measured" and simulated y-parameters was

$$c=\sum_{ij}\frac{|y_{ij}-\hat{y}_{ij}|}{|\hat{y}_{ij}|}$$

where y_{ij} is the y-parameter simulated from the fault model and $\hat{y}_{ij}$ is the "measured" y-parameter.

Table I shows the actual fault, the fault determined by the minimum cost function, and the value of the cost function of the correct fault. Of the seven examples at 1000 Hz, the algorithm correctly located five pairs of shorted nodes. The two errors seemed to be caused by the fact that the magnitude of the capacitor impedances at 1000 Hz were much lower than the impedance levels of the network resistances. (The magnitude of the impedance of a 10-μF capacitor at 1000 Hz is 15.9 Ω.) It was expected that the short circuit between nodes 6 and 7 would be difficult it identify, since the 50-μF bypass capacitor is only about 3 Ω at 1000 Hz, and so the changes in the parameters of the measured circuit could be expected to mask the short circuit. In fact, one of the reasons for this test was to determine how much the network parameters could be changed before this change prevented reliable identification of the short circuit. In this example, variations of the parameters of 10–20 percent from nominal still permitted correct identification of 5 out of 7 short circuits, or 71 percent of the cases tried. If the parameters of the "measured" network were set to the nominal values, then correct identification would occur, of course, in 100 percent of the cases, since then no error would exist between the "measured" and simulated y-parameters.

VI. Conclusions

We have shown an efficient way to analyze networks containing a single short circuit or other larger parameter change. The efficiency results from the fact that the network change caused by either the short circuit or other large parameter change results in an alteration of the inaccessible nodal admittance matrix by a matrix of unit rank. The response of the faulted network can therefore be computed in terms of the nominal response with many fewer multiplications than are required for and original analysis. Sparse matrix techniques were also used to reduce execution time.

The efficiency of the algorithm makes it practical to exhaustively search all possible single short circuits involving an inaccessible node in order to find a solution to the problem of locating a short circuit in an analog network from measurements at the accessible terminals. The test examples described in Section V show that short circuits can be reliably located by the algorithm, even though measurements were made at only one frequency and differences between the actual and the nominal parameter values were not taken into account. We expect that the reliability can be improved by making measurements at more than one frequency and by solving for the parameter values.

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